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AN INTRODUCTION TO 3D SPI2 (SPATIAL PACKAGING OF INTERCONNECTED SYSTEMS WITH PHYSICS INTERACTIONS) DESIGN PROBLEMS: A REVIEW OF RELATED WORK, EXISTING GAPS, CHALLENGES, AND OPPORTUNITIES

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ABSTRACT

Optimal 3D spatial packaging of interconnected systems with physical interactions (thermal, hydraulic, electromagnetic, etc.), or SPI2, plays a vital role in the functionality, operation, energy usage, and life cycle of practically all engineered systems, from 3D chips to ships to aircraft. These highly-nonlinear SPI2 problems, involving tightly constrained component packing, governed by coupled physical phenomena transferring energy and material through intricate geometric interconnects, have largely resisted design automation for decades, and can quickly exceed human cognitive abilities at even moderate complexity levels. Existing design methods treat the pieces of this problem separately without a fundamental systems approach and are sometimes too slow to evaluate various possible designs. Hence, there exists an emergent need to develop efficient SPI2 design automation frameworks for two reasons: 1) to enable the rapid generation and evaluation of candidate SPI2 design solutions; and 2) for the development of newer complex engineering systems. In this paper, the holistic 3D-SPI2 design problem with its attributes is defined, previous research efforts in various individual SPI2 related areas are reviewed, some existing critical gaps are outlined, and associated challenges are identified. Finally, a vision for fundamental research in SPI2 design based on the authors' experience in this topic is presented through a set of new exciting opportunities at the intersection of several engineering domains.

1 INTRODUCTION

A high demand exists in present society to create systems that are increasingly compact, while providing enhanced technical capability to realize benefits such as reducing emissions [1], increasing energy efficiency [2], and improving economic competitiveness. New SPI2 design automation methods are needed that can reduce the size of complex systems considerably, impacting applications such as power-dense smart batteries [1], spacecraft cooling systems [3], minimally-invasive medical wearables [4–6], vehicles with more usable volume [7], and compact avionics and military electronic systems [8, 9]. Engineers have labored for decades to improve spatial packing density across diverse domains such as in avionics [10–13], spacecraft systems [14], automotive packaging [15], vehicle electrification [16], and spacesuit design [17, 18]. These advances, however, have largely been incremental, and have depended heavily on expert human ingenuity. The more sweeping advances needed in SPI2 design and sophistication are hindered by the current lack of a unified SPI2 design theory and associated practical methods. Other engineering design domains, such as material distribution topology optimization (MDTO) and aeroservoelastic system design [19], have realized rapid progress in design capability and societal impact by successfully leveraging powerful design automation methods, such as design optimization, to help navigate design spaces that are too complex for expert human cognition alone.

Advancement in SPI2 system design will require similar formalisms and methods that do not yet exist. SPI2 design has been resistant to automation, in part, due to a lack of appropriate design representations for comprehensive SPI2 problems that are compatible with potential design automation strategies.

1.1 Objectives

The goal of this paper is to define the holistic SPI2 problem, review its constituent technical challenges, provide a vision for research teams to address these gaps in SPI2 design theory and capability, and catalyze the creation of powerful new SPI2 design methods and knowledge to take full advantage of the rich and complex design spaces associated with SPI2 systems. This will enable practicing engineers to go beyond what is possible using existing methods (usually based on packaging and routing design rules [20, 21], design heritage [22–24], and expert intuition [25, 26]) and 1) mitigate the costly packaging bottleneck in 3D system design, 2) enable a step change in the complexity of systems that can be optimally packaged, and 3) produce greater system performance and functionality, with much smaller footprints, by explicit treatment of complex design couplings through integrated design optimization methods. It must be noted that this paper serves as a preliminary attempt to consolidate different related aspects of the 3D SPI2 problem. The authors believe that there is significant opportunity for advances in SPI2 design knowledge leading to powerful SPI2 methods and tools, enabling a wide range of better systems.

The remainder of this paper is organized as follows. In Sec. 2, we define the holistic 3D-SPI2 problem, and its attributes. Its differences from 2D VLSI design are also discussed in detail. The different problem elements of SPI2 design research are discussed and previous related work in these areas is reviewed in Sec. 3. In Sec 4 critical gaps in SPI2 related research are provided. Section 5 presents the associated challenges in integrating the different SPI2 problem elements. In Sec. 6 a vision for SPI2 research with potential for significant impact is introduced, followed by a conclusion in Sec. 7.

2 SPI2 PROBLEM DEFINITION

The 3D Spatial Packaging of Interconnected Systems with Physics Interactions (3D-SPI2) problem can be defined as optimal spatial arrangement of heterogeneous geometric components and interconnects of often non-trivial sizes inside irregular three-dimensional volumes, along with the consideration of their physics-based behavior, life-cycle processes, and system operating conditions. These design problems cut across a wide swath of engineered-system domains that are vital to society (e.g., medical devices, transportation, and computing hardware), and entail especially large design spaces (combining complex combinatorial/topological, geometric, parametric, and time-dependent decisions) that are difficult to navigate either via expert human cognition or computational search. These have resisted holistic treatment by potentially powerful design automation methods, and still rely largely on manual, and sub-optimal spatial placement by designers supported by computer-aided design (CAD) tools. Solving the SPI2 problem requires highly skilled engineers who understand the engineering operation, manufacturing, assembly, testing, maintenance, and repair requirements. Moreover, design and maintenance of large-scale systems such as aircraft and ships requires thousands of man hours, while these systems' capabilities are unavailable, thus increasing the required sizes of fleets, and the associated cost. Any advancement to overcome this bottleneck has potential for significant technical and economic impact. The SPI2 design problem attributes are outlined as follows:

1. Fundamentally 3-dimensional; involves interconnected components with complex spatial geometries and often complex, irregularly shaped enclosing volumes;
2. Interconnects of various types (ducts, pipes and/or wires, etc.), sizes, shapes, and requirements (curvature, proximity, temperature, electromagnetic interference (EMI), etc.)
3. Strongly-coupled physics interactions (thermal, hydraulic pressure, electromagnetic, etc.) and influence of spatial arrangement on performance
4. Interconnect complexity (both topological and spatial), as illustrated in Fig.1
5. Unlike 2D systems, the 3D-SPI2 problem contains objects (components, casings, bays, etc) that are both solid and having holes or spatial-access ports. This makes the topological problem more complex. For example, an interconnect may pass through a hole in a component or bypass the hole and be routed between components.
6. Value metrics: spatial packaging density, volumetric power density, product life-cycle costs, system efficiency, system reliability, etc.
7. Constraints: geometric (for ensuring both feasibility and connectivity), physics-based, failure modes, etc.

2.1 Complexity of SPI2 vs. VLSI

Significant work has been performed in related areas, such as 2D VLSI (Very Large Scale Integration) circuit component layout, design and routing optimization [27, 28] for several board-based electronic applications. The VLSI design problem that has been automated successfully has limited degrees of freedom compared to more general SPI2 design problems. VLSI problems, in general, are primarily 2D applications with planar rectangular and other simple geometric surfaces [29] and do not involve extensive physics coupling between components and/or interconnect networks. Moreover, it is manually conceivable to design VLSI circuits both intuitively and by experience using existing design tools, and it is possible to estimate their efficiency or other related performance metrics in a systematic manner.

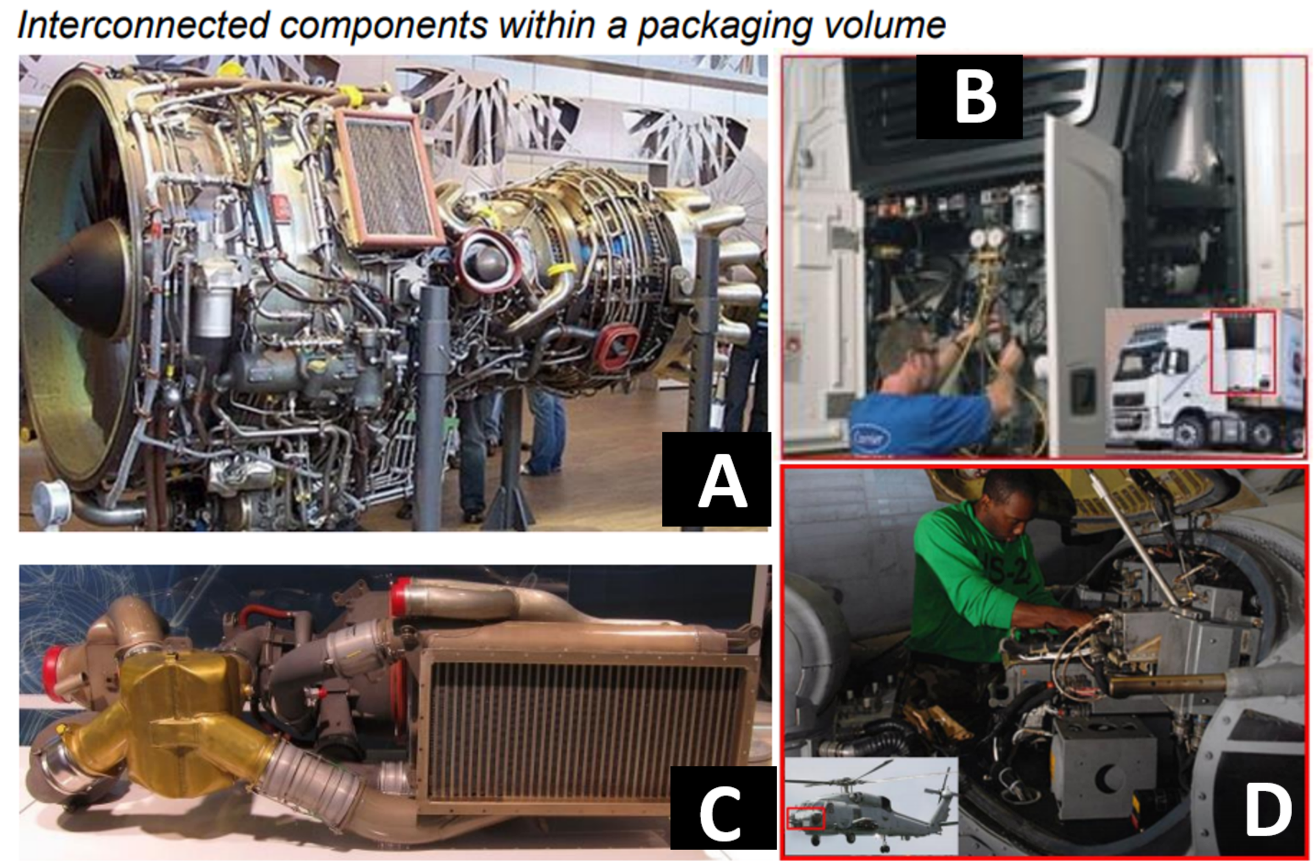

Figure 1: Diverse examples of typical manually designed systems presenting 3D SPI2 spatial packing and routing complexity, subject to physics interactions, and exhibiting spatial accessibility challenges for life-cycle processes: A) the externals (components, wires, pipes and ducts interconnecting components and engine features) of a commercial turbofan engine covering the limited surface area of its core and fan case, B) the refrigeration unit for a truck trailer, C) an environmental control system providing pressurization and cooling to commercial aircraft cabin air, and D) helicopter avionics hardware, interconnected by wire harnesses and thermal management pipes and ducts to reject electronics heat, presenting accessibility challenges in the front avionics bay.

In contrast, real-world SPI2 design problems are 3-dimensional (which adds a another layer of complexity) and have multiple diverse attributes such as components with complex spatial geometries (including concave and convex surfaces), restrictive domains, arbitrarily-sized, irregularly-shaped bounding volumes, interconnects of various types (pipes, ducts and/or wires, etc.) and radii, possible topological network configurations, strongly-coupled physics interactions (thermal, hydraulic pressure, electromagnetic, etc.), often large scale, and frequently encompass several other design challenges. Human judgment, even using available software tools, is insufficient to attain accurate, optimal designs or compare their size, weight, performance and cost. Another consideration is that SPI2 systems need to be designed with spatial accessibility to support safe and efficient manufacture, assembly, maintenance, diagnosis, overhaul, repair, upgrade, replacement, and complex operational requirements.

3 SPI2 PROBLEM ELEMENTS

The SPI2 design problem consists of different intricately related research elements that are individually very challenging themselves. The most important areas that are an integral part of SPI2 design research are shown in Fig. 2. The SPI2 problem can be subdivided into four basic sub-problems: 1) 3D packing, 2) 3D interconnect routing, 3) physics-based topology optimization, 4) SPI2 design representation. It should be noted, however, that there could be other similar domains such as evaluating various system life-cycle value metrics that might interact with these areas and could impact SPI2 design research. The focus here is on these four elements of research as they fundamentally address the SPI2 problem directly. State-of-the-art-methods and work done in each of these areas related to SPI2 design are reviewed in the following subsections.

3.1 3D Packing and Component Layout Design

3D component layout is a 3D bin-packing problem that comes under the class of mathematical optimization problems that involves optimally placing and orienting objects within a given 3D volume based on a set of value metrics, or optimally reducing the volume within which they can fit either alone or together with a set of value metrics. Packing problems are NP-hard. Typical engineering systems are a combination of functionally and geometrically interrelated components. The spatial location and orientation of these components affect a number of physical quantities of interest to the designer, engineer, manufacturer, and the end user of the product. The 3D component packing framework concerns itself with determining the optimal spatial location and orientation of a set of components given some design objectives and constraints. It models the layout problem as a volume minimization problem with the objective function being a weighted sum of the design objectives and penalties for constraint violation. The design objectives can include a quantification of a variety of measures such as the amount of cable used in the engine compartment of a car, the power density of an electronic component, the packing density of a drill, or the center of mass of a space vehicle. The most significant constraint is the non-intersection of components and non-protrusion of components outside the design space. Other constraints include spatial relationships between components (e.g. turbomachinery co-axially mounted on a shaft) and between a component and the packaging volume (e.g., gravity-based orientation of fluid reservoirs).

The 3D packing problem in most cases [30] is an optimal component placement layout problem, where component geometries can be arbitrary, with multiple types of design goals and spatial constraint satisfactions. For practical purposes, the minimization of layout cost functions is done under certain constraints imposed by design, fabrication and operational requirements. Most layout algorithms are restricted to a certain class of systems and are as a whole intractable due to their combinatorial nature. Problem variants differ by the particular definition of their packing constraints (presence of guillotine cuts, balancing and stability of the packing, possible overlapping of certain items, forbidden rotations of the items, etc.) and objective function, going by the well-known names of knapsack, bin packing, strip packing, variable-sized pellet packing, container loading, etc. Design automation methods for solving the optimal spatial packing problem have been developed and studied previously in the context of many applications, such as vehicle assembly [31], electronic module layout design [32], 3D container loading [33], bin packing [34], computer animation [35], the layout of components in additive manufacturing [36], and automotive transmission design [37]. Solution algorithms used in previous 3D layout packing research can be generally classified under four categories: genetic algorithms [38], heuristic methods [22], gradient-descent algorithms [39], and simulated-annealing algorithms [40].

3.2 3D Interconnect Routing

The 3D interconnect or pipe routing problem is a common industrial problem that is solved for designing layouts in chemical process plants, oil and natural gas refineries, water treatment and distribution plants, hydroelectric power plants, etc. Designing a 3D pipe layout involves two major tasks. First is equipment allocation, i.e., finding the 3D coordinate locations of all equipment to minimize total cost and satisfy all the equipment constraints such as maximum distances and maintenance access requirements. In this task, a rough measure is made to evaluate the total cost based on Manhattan distances. The second task is to find 3D routes for all the pipes connecting the equipment and to ensure that they are not colliding with each other.

Since the 1970s, interconnect routing design has been studied in various industrial fields, such as transportation, large-scale integrated circuits, and computing hardware. It is one of the most important processes for system integration. However, due to the complexity of routing systems and the diversity of constraints involved, it is quite time-consuming and difficult to achieve a feasible layout using both manual experience and CAD-based design tools. Systematic studies in route path planning have been carried out by researchers for several decades. Dijkstra's algorithm [41] proposed in 1959 is a well-known algorithm for path optimization with shortest length. Based on Dijkstra's algorithm, another heuristic algorithm was developed [42] to improve search efficiency. In 1961, Lee [43] proposed a maze algorithm to solve the problem of connecting two points. Further search efficiency developments have been presented [44,45]. Recently, research on route path planning has been promoted by the development of modern optimization algorithms such as genetic algorithms [46], ant colony algorithms [47, 48], and particle swarm optimization [20, 49]. For example, a genetic algorithm was used to optimize routing in three-dimensional space [46]. In addition, CAD-design based routing algorithms [21,50,51] were also applied in 3D pipe routing design.

In addition, many efforts have addressed the interconnect routing problem alone, where the component layout is fixed. Especially in the electrical engineering domain, many examples of 2D routing algorithms were developed for VLSI circuit layouts based on Manhattan rules and its variants [52]. Other 3D routing

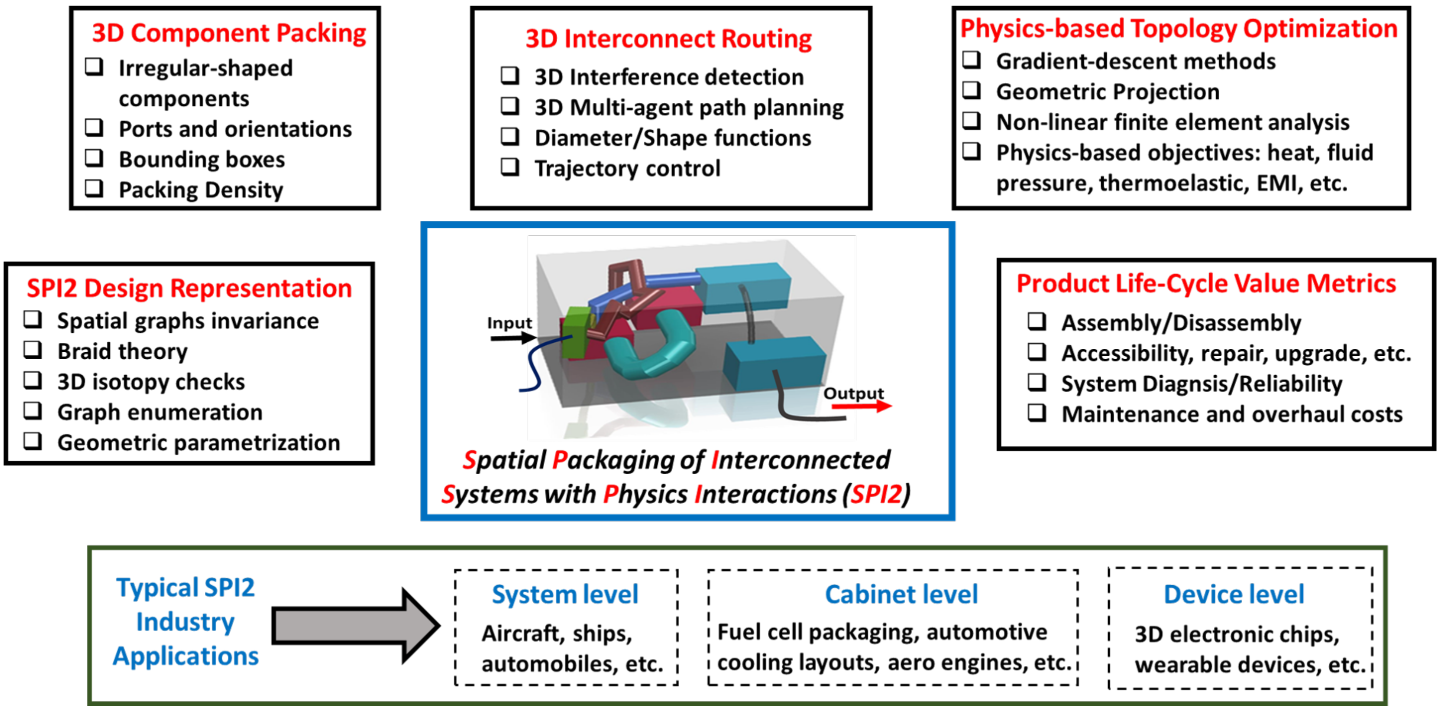

Figure 2: Primary SPI2 design research problem elements and some practical SPI2 industry relevant applications.

applications include aero-engine externals routing [53], ship pipe routing [54], chemical plant pipe routing [55], electrical wire routing in buildings [56], field-programmable gate array (FPGA) design [57], unmanned aerial vehicle navigation [58], and robotic path planning [59]. Optimization approaches have incorporated metrics such as packaging volume and mass properties [60], and have utilized solution methods such as simulated annealing [61, 62], pattern search [63, 64], genetic algorithms [65], ant colony optimization [66], and several other heuristic methods [24].

Finally, it is interesting to note that the 3D pipe routing problem, which aims at placing non-intersecting pipes from start locations to given target locations in a known 3D system environment, is very similar to the multi-agent path finding (MAPF) problem well known in robotics research. 3D MAPF research could directly serve as a potential field to address the 3D interconnect routing problem except that the former is dependent on dynamics of agents while the latter is a static problem.

3.3 Physics-based Topology Optimization

As mentioned earlier, an important aspect of the SPI2 design research is to integrate the physics interactions between the various components, interconnect flow passages, etc. as part of the geometric packing and routing optimization problem. Topology optimization, defined here as the optimal placement of material in a 2D or 3D geometric domain, does take into account models of physical behavior. This method class has been used across a range of engineering domains, including to design structures for maximum stiffness [67], multi-material properties [68], or component geometries for optimal heat conduction properties [69,70]. Problems that include multiple distinct physics domains have also been studied. De Kruijf *et al.*, Takezawa *et al.* and Kang & James performed optimization studies which included both structural and thermal conduction requirements [71–73]. The aerodynamic shape and internal structure of a wing have been optimized simultaneously [74–76] considering the interaction between aerodynamic loading and structural wing response. Topology optimization has also been used to optimize the placement of components and their supporting structure [77,78]. This allows sections of specific geometry, such as a pattern of bolt holes, to be distributed optimally within a structure. Designs produced by topology optimization are often infeasible for traditional manufacturing methods (subtractive, formative), but often can be made using additive manufacturing [79]. The design of components that are more easily manufactured using traditional methods motivates the development of methods that optimize designs made from standard material sizes and shapes, typically using ground structure methods [23, 80]. The geometric projection methods in Refs. [81, 82] have also been suggested to optimize structures made from stock materials.

Recent developments made in geometric projection method are highly relevant to SPI2 design research. An initial investigation by the authors of using projection methods for 2D SPI2 design problems can be found in Ref. [83]. The simultaneous physics-based packing and routing approach utilized in [84] makes significant system volume reduction possible. The projection method of Norato et *al*. [81] is extended to allow devices of arbitrary polygonal shape to be projected. Sensitivity analysis for this projection is provided to allow the efficient use of gradient-based optimization methods. These methods could be extended to model various combinations of physics; for example, fluid-thermal, thermal-electric or structural-fluid systems.

3.4 SPI2 Design Representations

Another very important aspect of any engineering design optimization problem is the design representation used for system modeling. Design representations must be accurate and compatible. For example, SPI2 representations must integrate with physics considerations, detailed geometric analysis, as well as navigation of formidable spatial topology decision spaces via graph-based enumeration. Spatial topologies involve how interconnects (such as ducts, pipes or wires) pass through or around other elements in a space, and these decisions represent one of the most difficult elements of SPI2 problems due to their combinatorial nature. Unlike 2D systems, 3D systems contain crossings and it is important to have representations that can support this need directly.

It must be noted that the 3D spatial packaging problem, even without considering physics aspects, is exceptionally difficult. In solving complex design optimization problems, much depends on the mathematical representations that are used to describe the various features of this system and system classes. To date, the authors have identified that three important mathematical representations 1) spatial graphs, 2) braids, and 3) homotopy classes can be used for representing 3D SPI2 design problems (as shown in Fig. 3) that are suitable for enumeration of different initial SPI2 topological designs that can be optimized geometrically. They are as follows:

1. **Spatial graph theory:** 3D engineering system networks can be represented as spatial graphs as demonstrated in our recent work [85]. Spatial graphs are graphs in three-dimensional space projected on a two dimensional plane. They are a natural extension of knot theory, which is the study of circles embedded in $\mathbf{R}^3$, since we can place vertices on a knot to transform it into a spatial graph. While the study of knot theory has its origin in the physics of the late 19th century [86], spatial graph theory has its roots in chemistry [87,88] and is different from graph theory because graph theory studies abstract graphs while spatial graph theory studies embeddings of graphs in $\mathbf{R}^3$ or even in other 3-manifolds [89–91]. This theory was used in polymer stereochemistry [87,92] and molecular biology (e.g., protein folding) to distinguish different topological isomers. A 3D system can be represented as a spatial graph where components are the nodes, interconnections are the edges, and the ports are node valencies.
2. **Braid theory:** Mathematical braid theory [93] can be utilized to represent the interconnect network within a 3D system. This allows efficient enumeration of various braid-based representations of the interconnect network, thus supporting the exploration of discrete topological system configurations. Braid and knot equivalence methods are leveraged to weed out redundant topologies. Braid and knot theory representations have been successfully used in other applications such as protein folding [94], and very recently in multi-agent motion planning [95, 96], etc.
3. **Homotopy classes:** Two trajectories are homotopic if one trajectory can be continuously deformed into another without passing through an obstacle, and a homotopy class is a collection of homotopic trajectories. Classification of homotopy classes in two-dimensional work spaces has been studied in robotics literature using geometric methods [97], probabilistic road-map construction techniques [98] and triangulation-based path planning [99]. There are many applications in robot motion planning [100] where it is important to consider and distinguish between different homotopy classes of trajectories (paths followed by robots). A strategy for classifying and representing homotopy classes in a 3-dimensional configuration space, using theorems from electromagnetism has been proposed [95]. BiotSavart's Law and Ampere's Law were used to define a differential 1-form, the integration of which along trajectories gives an invariant for the homotopy classes of trajectories. This concept of homotopy classes has been extended to defining different classes of 3D SPI2 problems into two categories: 1) systems containing only solid components (closing infinite or unbounded objects), and 2) systems containing both solid and hollow components (decomposing objects with genus > 1). Such mathematical extensions are very valuable in improving design richness and flexibility.

4 EXISTING GAPS

In this section, some of the most important gaps related to SPI2 design are outlined:

1. **Missing holistic treatment:** The main limitation with methods used in component packing, interconnect routing, and physics-based topology optimization is that existing efforts address these problems separately, rather than in a combined manner that accounts for inherent coupling. In addition, most of the methods consider only geometric aspects of the problem, neglecting important physical system proper-

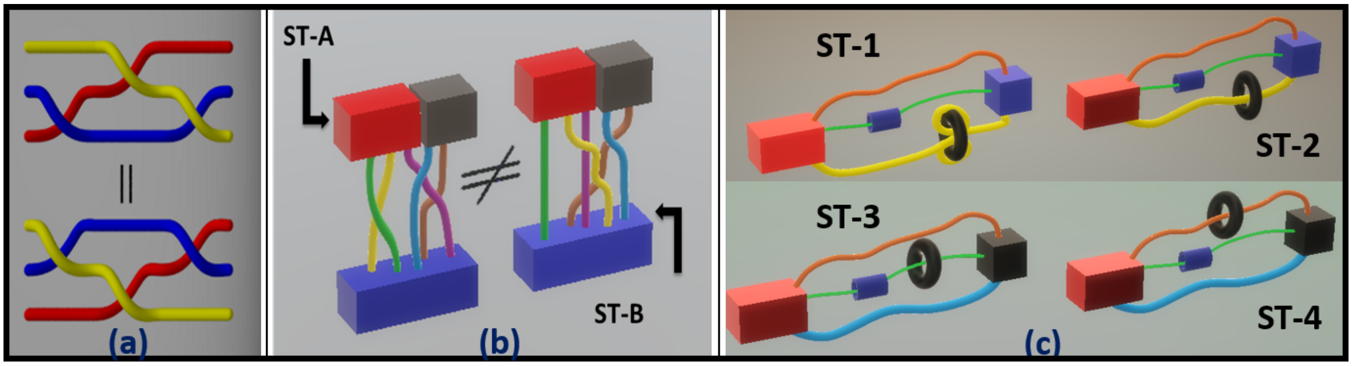

Figure 3: a) Two equivalent braids with corresponding braids words: [2 1 2], and [1 2 1] respectively; b) two different 3d system spatial topologies (ST)s enumerated using braid-based enumeration, and; c) four different STs of a system containing both hollow and solid components. ST-1, ST-2, and ST-3 come under different homotopy classes as the interconnects cannot be continuously morphed through the hollow objects they are passing through to attain the other 3D spatial topology. Note: ST-1 and ST-2 are different topologies due to the knotted yellow interconnect.

ties such as operating temperature/thermal loading, pressure drop, and aerodynamic and electromagnetic effects. Thus, existing methods may not extend well to the general coupled problem.

2. **Limits of manual design methods:** The amount of time required for a human designer to generate a feasible design and analyze its performance limits the ability of engineers to explore these complex design spaces within a constrained project timeline. Existing strategies can produce feasible designs, but they may not be optimal given all of the system requirements and design couplings, and the complexity of systems that can be considered is limited. In current practice, many aspects of the layout and routing problems are solved manually, which severely limits design capabilities for systems involving complex packing and routing tasks (especially in cases with strong physics interactions). In addition, the performance evaluation of the designs obtained from existing systems is left to human designers.
3. **Systematic enumeration of 3D topological design space:** A critical gap is the lack of methods to exhaustively search a SPI2 design space, such as those that have recently become available for system architecture enumeration. An efficient enumeration technique for navigating through the discrete 3D topology options possible for SPI2 design is required.
4. **Handling continuous and discrete elements together:** It is observed that this problem contains both continuous (spatial locations, interconnect diameter, trajectory, etc.) and discrete elements (topology options, number of components, interconnects, crossings, etc.). This is very challenging for optimization solvers and therefore there is a need for design optimization techniques that can efficiently navigate the combined space. In addition, unified geometric parameterization of both discrete and continuous variables should be performed to enhance optimization process efficiency and to aid improved problem formulations.
5. **Common design language:** SPI2 design research exists along the interfaces of several engineering domains and applications. To effectively communicate design knowledge between various communities of practitioners and domain experts, there is need for common terminology and constructs to address the problem elements.
6. **Visualization tools:** 3D SPI2 design problems have heterogeneous elements and this makes it challenging to conceptualize the design space. Both CAD-based and virtual reality based tools are required for viewing numerical simulations. Current tools are very limited with respect to SPI2 demands, and mostly serve the purposes of manual design activities.
7. **Human-centered design:** Human-informed design is required for developing better quality solutions to provide competitive advantage, improving end-user experience, increasing productivity and operational efficiency of product design pipelines, and for achieving greater system sustainability. Current SPI2 related research methods rarely bring human aspects to design which sometimes causes practical setbacks in industry adoption.
8. **Need for flexible design representations:** Existing design representations are developed to support specific-SPI2 related problems elements such as packing, routing, etc. and cannot be utilized for creating general design methods for holistic SPI2 applications. Therefore, there is a need to develop more unified representations that are both compatible for modeling and can capture the various SPI2 problem features with sufficient expressivity.
9. **Tailored SPI2 routing algorithms:** Existing approaches

widely use Manhattan rules for pipe routing, but improved SPI2 system performance requires flexible, and maybe deformable, pipe-shape routing representations depending on applications. In addition, optimal trajectory control of pipe routing has not yet been performed in the existing interconnect routing research. Achieving this capability could help satisfy several practical SPI2 constraints. For example, if a pipe should pass through some way points or physical fields on its path for several practical reasons such as cooling a hot component, fluid transfer, etc., then optimal trajectory control plays a vital role.

5 ASSOCIATED CHALLENGES

The associated challenges related to SPI2 research are identified as below:

1. Both packing and routing are NP-hard problems. Therefore, as the scale and complexity of the system increases, the number of possible solutions explodes combinatorially, increasing decision-making cost significantly.
2. The 3D topological space is vast and challenging to navigate as there can be infinite design options depending on the tuning parameters. Therefore, it is essential to have sampling strategies that can cover the design space thoroughly and efficiently.
3. The 3D-SPI2 problem is a highly nonlinear optimization problem simultaneously trying to address packing, routing, and physics performance evaluation. Therefore, there is a greater possibility of local solutions with continuous spatial or parameter tuning when compared to design optimization of individual SPI2 problem elements.
4. One key challenge in using gradient-based solution methods, such as the geometric projection method [81, 84], is that changes in interconnect spatial topology may impact the lumped-parameter system models (such as fluid loops) in ways that either prevent simulation of certain designs, or at a minimum introduce non-smoothness.
5. Creating design representations that can support topology, geometry, and physics aspects of the 3D SPI2 problem in a unified way is one of the most challenging aspects. Conventional methods address at most a pair of them while solving multi-physics optimization problems. Previous work exists where all three aspects are included but they are specific to their applications and not general.
6. SPI2 design automation tools should also consider the human perspective in all steps of the problem-solving process. In particular, industry practitioners who have vast experience in handling these complex systems possess valuable design knowledge that can be applied while developing SPI2 design automation frameworks. Incorporating human expertise into SPI2 automation methods, however, may introduce human biases or errors.

6 DISCUSSION

3D component packing and 3D routing problems are individually NP-hard problems and solving the combined problem with multi-physics interactions and couplings between system elements is thus especially challenging. The packing and routing problems, however, should be solved simultaneously to achieve system-optimal designs. A sequential effort, such as pack-then-route or vice versa, cannot fully exploit design coupling between all sets of decisions. The challenges are growing in significance as system compactness and performance requirements intensify. For example, commercial aircraft engines a few decades ago were larger compared to current designs; modern aircraft engine cores have a much smaller diameter and thus surface area, but must incorporate essentially the same externals, such as wires, pipes etc., as older designs.

6.1 SPI2 Vision

The broader goal of this research is to not only create methods to solve the SPI2 design problem, but also attempt to answer some larger SPI2 design research questions, identified as follows:

1. How to characterize the SPI2 design space: What are its boundaries? How do the feasible and infeasible design space regions compare with each other? What optimization methods are require to navigate the design space and estimate what of that space is explored?
2. What design optimization frameworks are required to integrate different SPI2-related research areas together, search the SPI2 design space effectively, aiding both discrete and continuous decision making.
3. Is the SPI2 design space generic, or does it need to be classified according to system size (e.g., number of components), complexity, physics performance requirements, product life-cycle cost value metrics, or other dimensions?
4. How does SPI2 design difficulty scale with increase in the number of components, constraints, complexity, etc?
5. What kinds of design tools does industry require to adopt SPI2 design automation methods?
6. What unified design parameterizations/representations are needed to solve the SPI2 optimization problems efficiently?
7. How might various system-life cycle value metrics such as manufacturing, maintenance, upgrade, overhaul, repair, and accessibility costs be incorporated as part of the SPI2 problem formulation and automated solution?

6.2 A Two-stage SPI2 Design Automation Framework

Rather than make incremental progress on established methods for spatial packing and routing (PR), we recently developed a novel two-stage design framework [83] for solving SPI2 problems using a continuous representation. The continuous representation enables the use of gradient-based methods to efficiently search the packing and routing space. This methodology is centered on the use of simple geometric bars to approximately model both the component geometry and routing paths. Bars have favorable geometric properties that can be exploited to represent both the packing and routing problem, and solve them simultaneously. This simple geometric representation was used in two stages to perform physics-based packing and routing. More complex shapes have been incorporated after the basic method was established. The success of this method has been demonstrated for simple 2D PR test cases. In Stage 1, unique spatially-feasible spatial topologies are enumerated for an electro-thermal system [101]. It is important to guarantee a feasible initial graph as lumped-parameter physics analyses may fail if components and/or routing paths intersect. In Stage 1, one of three strategies is used to generate interference-free initial layouts: force-directed layout, shortest path routing, or topology enumeration. Curved bars are used to represent both components and interconnects. Stage 2 begins with a spatially-feasible layout, and optimizes physics-based system performance with respect to component locations and interconnect paths [83, 84]. The bar-based design representation enables use of a differentiable geometric projection method (GPM), where gradient-based optimization is used with finite element analysis. Differentiable GPMs have been used in several domains, including moderately high-dimension structural design [81]. Barrier functions can be applied to prevent component/interconnect interference implicitly. A core challenge with extending this effort to 3D PR problems involves a transition to 3D GPMs, which use plates as geometric primitives, whereas 2D GPMs use bars. Optimization should be performed with respect to plate location, shape, and orientation parameters as opposed to discretized design representations (e.g., element-wise densities or node-wise level set values of conventional topology optimization approaches). This approach also has the benefit of simplifying treatment of geometric constraints.

The 3D SPI2 problem, however, involves multiple discrete options for geometric topology in comparison to the 2D routing problem. More specifically, even if the system architecture remains fixed (how components are connected), options exist for how the interconnects are routed relative to each other These are fundamentally discrete design options. One key challenge in using the gradient-based solution methods is that changes in interconnect geometric topology may impact the lumped-parameter system models (such as fluid loops) in ways that either prevent simulation of certain designs, or at a minimum introduce nonsmoothness. We have identified multiple promising strategies for managing interconnect topology decisions, and plan to explore these options in conjunction with the continuous aspects of the problem. One strategy will be to utilize efficient graph-based enumeration strategies [102–104] to enumerate feasible interconnect topology options, and then for each option, solve the continuous optimization problem. This has potential for scaling to large systems using machine learning trained on enumeration data. The second strategy is to use penalty functions and other possible topology optimization techniques to allow interconnects to pass through each other while preserving model smoothness. In this way, we can relax the discrete topology design problem, and absorb this task into our broader gradient-based design framework. Steps also need to be taken to assess and mitigate additional challenges such as local minima, which can arise when implementing relaxation methods.

7 CONCLUSION

Effective design automation strategies are key to meeting the demands of present and future needs for 3D physics-based system packaging problems. Systematic, flexible, and efficient design methods with the ability to explore and access new configurations are essential for achieving better system performance, compactness, and life-cycle cost, across different engineering industries. Effective methods will support adjustments that can be made easily as the system requirements change over time. An important potential benefit of realizing such methods is the reduction in design time and resources required to solve packing and routing problems, enabling greater tailoring of designs to enhance performance for unique applications, while reducing design effort.

Creating a body of knowledge within the 3D packaging space is central to solving important problems throughout the product life cycle, from manufacturing, to assembly, maintenance, diagnosis, repair, and retrofit. Simple designs are typically employed to keep these problems tractable; providing a means to reason in this complex space offers an unprecedented opportunity to increase product performance and packaging density, while leveraging advanced manufacturing methods and automated assembly methods. This paper reviews the technical groundwork, defines the shape and bounds of this knowledge domain, and specifies an initial set of key areas to jumpstart the engineering research community's efforts in this field. Some of the existing critical gaps that prevent the creation and successful application of design automation methods to industry-relevant holistic SPI2 problems are outlined, and associated challenges are addressed. Finally, some larger SPI2 design research questions are presented and one approach developed by the authors is discussed as an example of handling SPI2 research problems. This paper is considered to be an initial introduction of the SPI2 class of problems to promote discussion and catalyze a surge of research activity in this domain. In the future, these topics will be demonstrated in more-depth with illustrative examples, representation, and solution methods.

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