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HYBRID PROCEDURE-BASED DESIGN STRATEGIES AUGMENTED WITH OPTIMIZATION

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ABSTRACT

Procedure-based design is well-established, supporting engineers via expert knowledge codified in resources such as handbooks, tables, and heuristic if-then rules of thumb. These procedures enable even inexperienced designers to benefit from the knowledge obtained by more experienced counterparts through years of practice and discovery. While procedural approaches have many advantages, they do have limitations. They tend to produce only satisficing, rather than optimal, solutions. In addition, they are based on historical designs, so offer little assistance for new system types, and are often descriptive rather than normative in nature. In contrast, normative methods—such as constrained optimization—can resolve many of these issues, but at the cost of significant development effort. Here we present a synergistic hybrid strategy with the objective of capitalizing on established procedure-based design methods for a subset of design problem elements, while incorporating normative strategies for the remaining elements. A design procedure is analyzed to identify steps that involve specification of design variables, and a subset of rule-based steps that could be replaced with optimization algorithms. A single-stage spur gear train design example is used to illustrate this process, and for comparing alternative hybrid solution strategies. Initial results indicate that solution quality can be improved significantly over purely procedure-based design when incorporating limited optimization elements, while maintaining a reasonable level of additional modeling effort.

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1 INTRODUCTION

Engineering design is defined as the use of scientific principles, technical information, and imagination in the definition of a mechanical structure, machine, or system to perform prespecified functions with the maximumal economy and efficiency [1]. A primary task of engineers is to apply their scientific and engineering knowledge to technical design problems, and to optimize solutions within the requirements and constraints set by material, technological, economic, legal, environmental and human-related considerations [2].

Engineering design often requires a systematic approach involving objectives, variables, and limitations (technical, social, monetary, time and other resources) as inputs, and generates an output with the aim to 'change an existing situation into a preferred one' [3]. When defining design decision variables, the designer must identify which parameters can be controlled directly [4]. Objectives should reflect design intent. One strategy for identifying promising design concepts is to review previous designs that have succeeded in achieving similar objectives. In established engineering application domains, years of experience and design heritage are available to support future design efforts. Often the knowledge and expert intuition resulting from this experience is archived in a useful, reusable manner. Design rules and procedures (with varying degrees of sophistication) can be defined and used that capitalize on this knowledge. Early efforts to encode design principles into computer-based design tools revealed a central challenge: deciding which aspects of a design process as practiced by human experts to retain, and which to replace with design automation strategies [5].

Archived design procedures are often developed with the objective of simultaneously improving design success and reducing design effort. These procedures may be found in handbooks, design codes, or in many cases proprietary documentation. While these procedures often help in generating an acceptable design (including observation of practical considerations), they normally do not lead to highly improved or optimal designs. Another limitation that decisions are almost always coupled, and sequential procedures cannot fully account for interactions between decisions. One resolution is to decompose a problem into temporarily decoupled subsystems, but this can affect alignment between subsystem design efforts and improving system utility due to narrow focus. The problem of biased information from subsystem designers leading to system sub-optimality is addressed in Ref. [6], where a subsystem incentive structure is proposed to improve robustness against biases. Such systematic design procedures aim to steer the efforts of designers from unconscious into conscious and more purposeful paths with overall system design in mind [2].

Normative design approaches, such as design optimization, require articulation of design decision variables and their controllable ranges, identification of formal objective and constraint functions for quantitative comparison of design alternatives, and development of accurate models that predict the effect of design variable changes on objective and constraint functions. Developing models that are appropriate for design optimization requires a deep understanding of the application, the underlying physics or other governing phenomena, and the nature of optimization solvers to be used. While design optimization can lead to significant performance improvement, in some cases implementation cost may be too high [4].

This decomposition approach is also employed in [7], where the design process is broken down into a series of decisions, and models of varying fidelities are constructed. First, low-fidelity models were employed to decrease the size of the search space, then higher-fidelity models which are more accurate (but also more expensive) are employed to improve the solution. Model-based design efforts are impacted by the properties of the underlying mathematical models [8]. Some limitations of normative design methods include: 1) the processes used to achieve designs are more removed from the way humans carry out these tasks, and 2) much of design can only be represented symbolically but not mathematically.

The premise of the ideas presented here is to create unique hybrid solution strategies that transition from procedure-based design, a descriptive design method, to optimization, a design strategy that could be considered normative (acknowledging that it requires simplifying assumptions that imply it has heuristic properties). The idea is to lead designers to favorable solutions quickly and intuitively through logical modification of existing expert rule-based sequential design strategies by leveraging optimization in a limited way. Hybrid approaches show promise, but

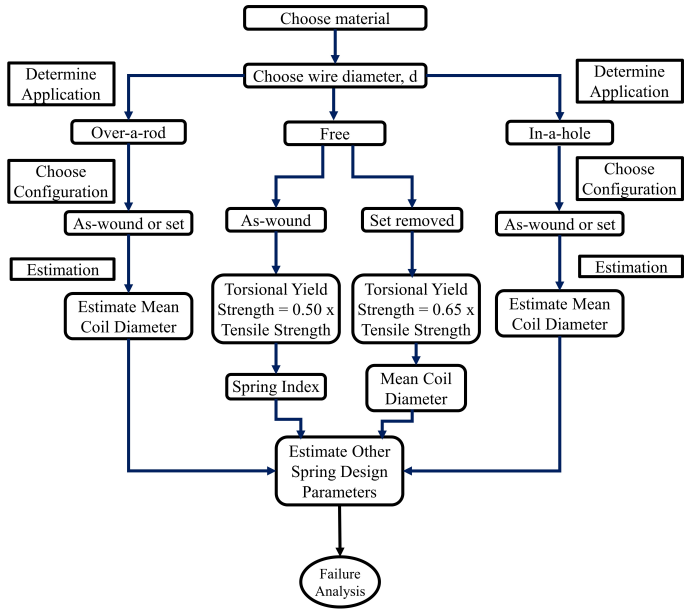

FIGURE 1. AN EXAMPLE OF PROCEDURE-BASED SPRING DESIGN [11]

pose the new problem of determining how to accomplish the hybridization. The problem of determining the best combination of procedural and normative design of decision support systems is addressed in [9]. They employ empirical methods for determining the best approach, constructing and testing alternative combinations. Both procedural heuristics and normative probabilities are used in [10] to rank a set of alternatives ordinally and analyze only the top subset of "good enough" or "satisficing", alternatives.

Procedure-Based Design involving expert heuristics, design rules, and flowcharts is discussed in the first subsection of the introduction. This is followed by a review of modeling complexity, effectiveness, advantages and drawbacks of Optimization-Based Design. Based on these discussions, a Hybrid Design Strategy is proposed in Section 2. An illustrative example, involving a single-stage spur gear train design problem, is used as a basis to demonstrate and evaluate hybrid strategies. This example is introduced in Section 3. Conclusions and future work are examined in the final section.

1.1 Procedure-Based Design

A design methodology can be defined as a concrete course of action for the design of a technical system that relies on knowledge from areas such as design science, cognitive psychology, and from practical experience [2]. A design handbook often includes design strategies in the form of sequential instructions, often involving quantitative rules. Here we refer to this type of strategy as Procedure-based Design (PBD).

PBD incorporates expert knowledge based on extensive experience with a particular type of system, design rules derived from empirical data, and pertinent scientific knowledge organized logically into a form that less experienced engineers can use to support their efforts to generate successful designs, and that can also be useful for experts. These design procedures embody expert domain knowledge efficiently, and eliminate the need to formulate a formal mathematical model of the design problem.

Design procedures often incorporate an expert heuristic rule-based system, as described in [12]. A common format is a flowchart representation with conditional statements, similar to diagnosis flowcharts used by technicians repairing systems. Often these flowcharts, when followed precisely, help engineers develop a feasible design even with limited domain knowledge within a few design iterations. An excerpt taken from the synopsis of Dudley's Handbook of practical gear design describes it as a 'Handbook that requires limited knowledge of mathematics for adequate application to almost any situation or question' [13]. Figure 1 illustrates an example of PBD for mechanical spring design, given design requirements [11]. Most steps involve calculations based on physics-based analysis or empirical relationships.

PBD can be time and resource efficient, and often is a robust strategy for generating usable designs. Handbooks often include heuristic models derived from empirical data, adjustments for manufacturing standards and other recommendations to address practical considerations. These handbooks are often more accessible to practicing engineers than journal articles that provide the foundation for PBD. One important drawback is that PBD is often based on expert opinion, which may vary across experts. The impact of this variation has been a topic of investigation [14]. One particular PBD method, the change prediction method, was shown not to be robust with respect to expert variability, leading to reversals in the rank order of risk values for change propagation.

PBD methods can also introduce certain inherent cognitive biases that can sometimes inadvertently result from the use of expert heuristics [15]. Solutions obtained from PBD, although often provide feasibility guarantees, may have an unclear path towards optimality. While PBD often can produce a 'good-enough' solution, such methods often evaluate only a small subset of all possible solutions, and are inherently limited in design exploration. The initial solution for a design problem takes a small number of iterations, but this number increases significantly if the objective is to work toward an optimal solution. A design expert, however, can reduce required iteration by applying deeper expert intuition not captured in the PBD method, in part by internalizing complex cause and effect relationships within the paradigm of a conditional rule-base. A novice designer, on the other hand, will lack this type of intuition. They can follow procedures but may develop little insight into the knowledge that helped formulate the PBD approach.

For many design activities, satisficing [12] may be the only goal, but this paper introduces a strategy for moving beyond an initial "good enough" solution to one that may be significantly closer to the optimal, while limiting increase in design effort.

1.2 Optimization-Based Design

Formulating an engineering design problem as a mathematical optimization problem is an established strategy for modeling design problems [16]. A standard formulation, in negative-null form, is:

$$\begin{aligned} \min_{\mathbf{x}} \quad & f(\mathbf{x}) \\ \text{subject to:} \quad & g(\mathbf{x}) \leq 0 \\ & h(\mathbf{x}) = 0 \end{aligned} \tag{1}$$

Where $f(\mathbf{x})$ is the objective function that is to be minimized (e.g., cost, performance metric, etc.), $g(\mathbf{x})$ are inequality constraints that bound designs away from failure (e.g., stress, temperature, packaging etc.) or other limitations, and $h(\mathbf{x})$ are equality constraints. All of these functions depend on the design vector $\mathbf{x}$, which is a vector of independent design variables that are to be adjusted during the optimization process. The design vector represents the design in an abstract way. In this formulation we assume that $\mathbf{x} \in \mathbb{R}^n$, but other formulations admit discrete values for $\mathbf{x}$.

Design optimization effectiveness depends on both how well the formulation reflects design intent, and on how accurately we can predict the effect of design changes on system behavior using a mathematical or computational model. The solution to a design optimization problem does not generate the optimal design for the real product, but is an approximation of the true optimal design (which is often unreachable) [17]. The solution is optimal only with respect to the formulation and predictive model used.

Gradient-based optimization algorithms are often preferred due to numerical efficiency, but, except for limited exceptions, can identify only local and not global optima [10]. The probability of finding a global optimum can be improved by solving the problem multiple times with different starting points. Some gradient-free algorithms, such as genetic algorithms, can improve the probability of finding a global optimum, but global optimality cannot be proven. One may simplify a formulation and corresponding model using techniques such as linearization to produce a problem for which global optimality can be proven, but such simplifications move us farther from the solution to the original design problem. In practical engineering applications it is often better to use a more complete formulation and model for which we can only approximate the global optimum.

One can argue that setting up a design optimization problem is a non-sequential approach to solving a design problem.

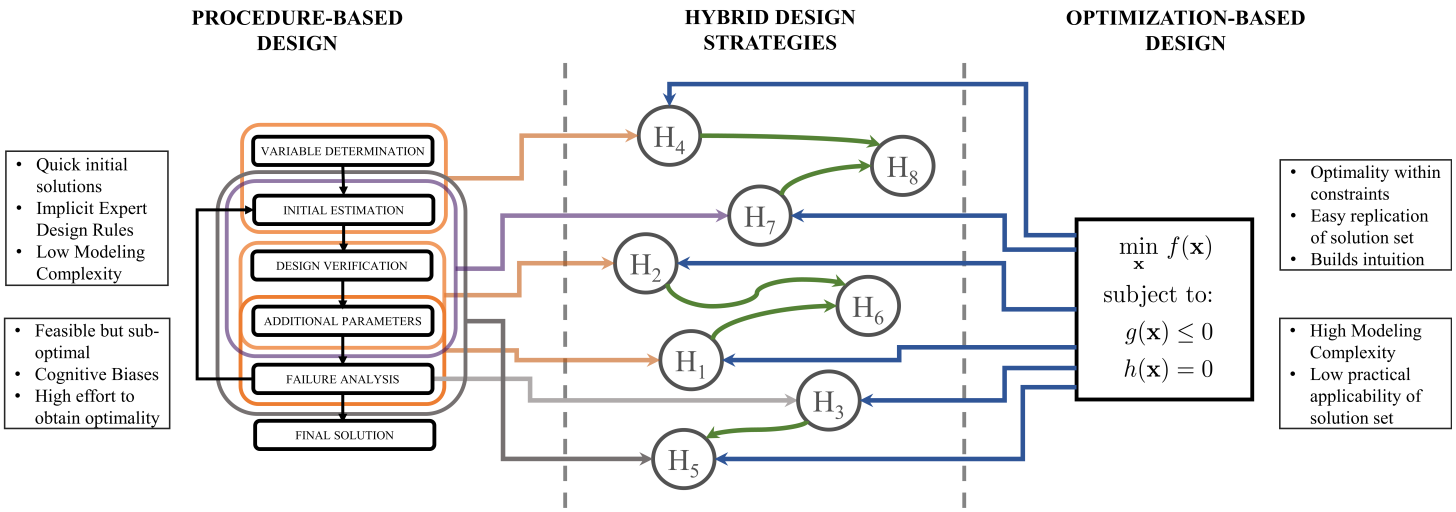

FIGURE 2. CONCEPTUALIZATION OF THE DEVELOPMENT OF HYBRID STRATEGIES

It is based on collection of precise models, specification of correct constraints and design decision variables, and then using a computational tool to solve the problem. This accentuates the need for the designer to understand deeply the application, optimization theory and algorithms, and system modeling. Solving a practical design problem using optimization alone is often very effort-intensive, and engineers must evaluate whether this level of investment is worth the potential performance increase. It may be especially difficult to incorporate explicitly all practical design considerations into a design optimization implementation that are accounted for in PBD, especially for very mature engineering applications. This issue will be explored further using the illustrative example.

2 HYBRID DESIGN METHODS

Here a design method is introduced where optimization subproblems are solved within a PBD framework. This strategy creates multiple clusters of new solution sets. It utlizes expert opinion and empirical knowledge, while leveraging optimization. Using hybrid design strategies require some additional prerequisite knowledge. Here we assume that a single objective function can be identified, and is used consistently through the design activity.

Understanding the relevant implicit design rules, especially those with far-reaching consequences for the final solution set, requires thorough understanding of the intrinsic system complexities and possible non-linearities. Design rules based on extensive experience often capture these complexities well. As we begin to explore hybrid design strategies, we start by retaining rules that manage more complex design elements, and replacing more straightforward rules with optimization-based decisions (as simple as a single design optimization variable).

PBD approaches often include conditional statements at particular stages for checking satisfaction of some constraints, sometimes in an indirect way. The final solution is checked for complete compliance with design constraints usually only at the end of the design procedure. In a hybrid strategy some of the constraints may be incorporated into optimization formulations, particularly when constraint functions depend on variables being determined via optimization. Decisions made by solving optimization subproblems are then inserted back into the PBD sequence, replacing a rule-based decision element.

For each iteration of a hybrid solution strategy, the design variables not determined via optimization can be varied by human judgment within their relative bounds and based on PBD rules. The solution set obtained from multiple iterations can reveal crucial information about the relationships between the chosen optimization variables with other variables and impact on system behavior.

Hybrid strategies may be adjusted in a sequential manner, where additional design decisions can be added to the set of decisions made via optimization rather than PBD. Figure 2 illustrates a conceptual approach for constructing hybrid design strategies, showing that some decisions are made based on PBD rules, and others using optimization. In this study we explore multiple hybrid strategies, leading toward future investigations where rigorous guidelines are developed for hybrid strategy construction. One possible approach for exploring different hybrid strategy options is to explore augmentation with single-variable optimization problems, and then consider multi-variate optimization problems. In some cases, especially when design variables interact, multi-variate optimization can be especially effective compared to human judgement.

Increasing the number of design variables comes at the cost

of increasing modeling complexity for the constraints and objective function, increasing non-linearity, and an overall increase in effort. This increase may not be proportional to hybrid strategy usefulness. Thus, after the creation of hybrid solution strategies, a retrospective multi-attribute utility analysis is suggested on the available and hypothetical design strategies to determine whether or not the trade-off is worthwhile. The creation of new strategies may also be limited by available resources for modeling and computation.

3 ILLUSTRATIVE EXAMPLE: SINGLE STAGE SPUR GEAR TRAIN

Gear design is a complicated, but well-understood design activity. A variety of approaches to obtain optimal gear design have been introduced [18–23]. A single-stage spur gear train consists of two meshing spur gears on parallel shafts. Three design variables, namely, number of pinion Teeth N_p, diametral pitch P_d (interchangable with module), and face width F are selected here as the design representation. The following are fixed parameters used in gear train analysis:

1. Power of the motor connected to the pinion shaft (in HP)
2. RPM of the motor connected to the pinion) shaft
3. Required speed ratio (m_g)

Material parameters corresponding to carburized and hardened steel are chosen for both gear and pinion.

3.1 Design Objective

The design goal is to create a compact gear set for a set of fixed input parameters that meets failure and manufacturability requirements. Carroll [20] notes that a compact gear set is easier to manufacture, runs smoother due to smaller inertial loads and pitch line velocities, and is less expensive. It may also be beneficial from a packaging perspective for some applications. The objective function is defined here as a sum of two individual functions. The first is based on geometric dimensions:

Gear Set Size:

$$f_1(\mathbf{x}) = d_{o,p} + d_{o,g} \tag{2}$$

where $d_{o,p}$ is the outer diameter of the pinion (driving gear), $d_{o,g}$ is the outer diameter of the gear (driven gear), and $(\mathbf{x})$ is the vector of design decision variables. The second objective is an estimate of gear mass:

Estimated Gear Set Mass:

$$f_2(\mathbf{x}) = \frac{\pi}{4}(d_{p,p}^2\rho_p + d_{p,g}^2\rho_p)F \tag{3}$$

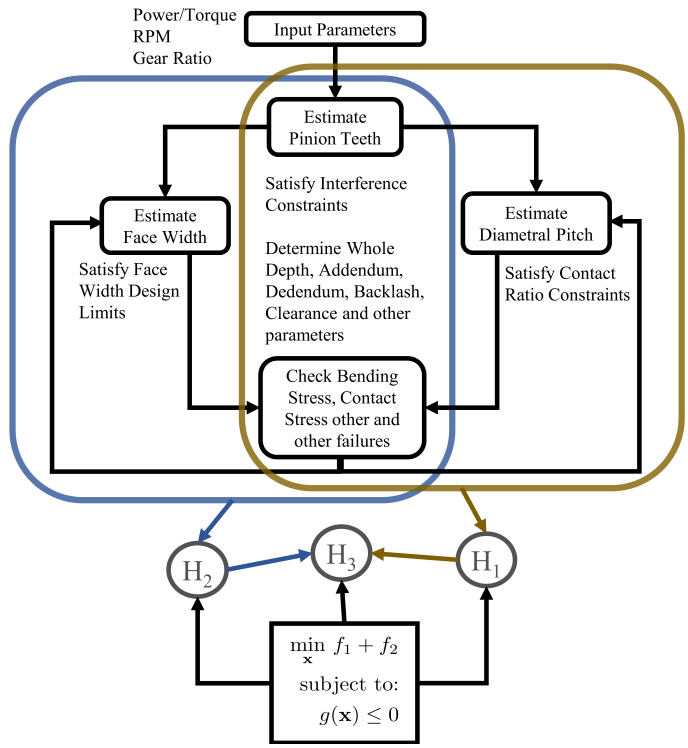

FIGURE 3. DEVELOPING HYBRID STRATEGIES FOR DESIGNING A SINGLE STAGE SPUR GEAR TRAIN

where $d_{p,p}$ and ρ_p are the pitch diameter and the pinion material density, respectively, and $d_{p,g}$ and ρ_g are the pitch diameter and density of the gear material, respectively. F is the face width. The two functions are summed to form the overall objective function:

$$f(\mathbf{x}) = f_1(\mathbf{x}) + f_2(\mathbf{x}) \tag{4}$$

Since the two objectives lie on the same scale, a rather simplified objective function is used. More sophisticated approaches may include objective function scaling, weighting based on system utility assessments, or multi-objective tradeoff studies.

Detailed design of single stage spur gear train was carried out first using an established PBD method, followed by hybrid and purely optimization-based strategies. This supports a comparison and analysis of the different strategies. Several practical considerations and constraints were included in the analysis. Comparative manufacturability of results from the different design strategies was also studied.

3.2 Procedure-Based Gear Design

The Second Edition of Dudley's Handbook of Practical Gear Design and Manufacture [13], published first in 1954, is a well-recognized and extensive handbook for supporting gear design.

The PBD strategies for spur gear design in this handbook were adopted here. Other resources for gear design were also used in this development [11, 24].

Dudley suggests that the easiest way to design gears is first to estimate required gear size. If the designer can correctly estimate required size, the remaining steps involve straightforward calculations and adjustments after checking the design with appropriate rating formulas. Since our goal here is compactness, we omit Dudley's size estimation strategy. Instead, we apply the procedural design approach depicted in Fig. 3, with the intention of obtaining the minimum size and mass as quantified in Eqn. (4). A ten-step procedure was implemented using MATLAB® to automated the PBD strategy, mimicking appropriate use of conditional statements as specified in the handbook. This automated strategy is intended to approximate designs that may be generated by a human designer using the same PBD.

The design procedure begins with specification of application requirements (power gearing, speed gearing etc). This information is used to identify an appropriate value for pressure angle, ϕ, which is commonly set at $\phi = 20^\circ$ in commercially available gears. The next step is to choose an initial value for the number of pinion teeth, N_p. This is identified as an early example of where expert intuition can lead to an improved design decision. Intuition based on extensive experience can help a designer choose a good value for N_p at the start, reducing required iterations. Alternatively, a designer can utilize a reference table to obtain suggested values for initial N_p depending on the problem. Next the designer assesses whether the result is applicable 'for a good balanced design'. Different handbooks [11, 24] suggest different criteria for choosing the right estimate for number of pinion teeth but the values are usually similar.

The next step is to determine approximate center distance and face width values based on N_p. The center distance is:

$$C = \frac{1}{2P_d}(N_g + N_p) \tag{5}$$

where N_g is the number of gear teeth.

These design variables interact, and it is difficult to determine appropriate values for all of them simultaneously by referencing tables or the available equations. Other unknown variables complicate this tasks. Inspection of the objective function suggests choosing a smaller face width value to reduce mass, but is bounded below by material and manufacturing process limitations. These bounds are non-obvious in the context of other variable interactions. Center distance bounds may be estimated by analyzing the maximum available space and shaft sizes. PBD estimates are appropriate for processes focused on finding a satisficing design, but pushing system performance limits using optimization requires more rigorous specification of these limits.

Dudley suggests determining the diametral pitch based on previous estimations, and then readjusting for the possible use of a standard pitch. It is important to note that the diametral pitch places a requirement on the manufacturing tool and tool size. In many cases it may turn out to be advantageous and resource-efficient to use available standardized tooling, but at the cost of reduced technical performance.

TABLE 1. RESULTS OF PROCEDURE-BASED DESIGN. THE PBD AS PRESENTED IN DUDLEY'S DESIGN HANDBOOK [13] WAS MODELED AS A MATLAB PROGRAM, AND DESIGN WAS CARRIED OUT TO ACHIEVE OPTIMAL OBJECTIVE FUNCTION VALUE. THE INITITAL RESULTS FROM PBD WERE FOUND TO BE SIGNIFICANTLY WORSE THAN EARLY DESIGN SOLUTIONS FROM HYBRID DESIGN STRATEGIES.

Total Number of Iterations	P_d	F	N_p	Objective Function Value
10	2.5	4	16	1034.805
11	3	4	16	729.698
14	4	3.5	17	418.44
15	4	3	17	364.806
16	4	2.5	17	311.172
19	4	2.5	18	346.149
20	6	2.5	18	163.955
28	8	1.5	25	118.493
29	8	1.5	26	126.842

Following the PBD process correctly generally leads to a feasible design specification. The loading capacity for each resulting design is then checked for bending and contact stresses as defined by AGMA (American Gear Manufacturers Association), illustrated in Eqns. (6) and (7), respectively:

$$S_t = \frac{W_t K_o P_d K_s K_m K_v}{FJ} \tag{6}$$

$$S_c = C_p \sqrt{\frac{W_t K_o K_s K_v K_m C_f}{d_{p,p} F I}}, \tag{7}$$

where K_o, K_v, K_s, K_m are the overload, dynamic, size, and load-distribution factors. C_p and C_f are the elastic coefficient and surface condition factors, I and J are the geometry factors, and W_t is the tangential load.

Table 1 lists the results obtained from PBD. Within nine iterations, a feasible design can be obtained. The resulting objective function value, however, is far from the optimal (as detailed soon). Extended analysis of the underlying equations and

assumptions can help improve design performance, but requires greater experience and effort.

PBD produces a design that satisfies intrinsic stress and physical (contact ratio, tooth deflection etc) constraints with relatively low design effort. The guidelines and conditions provided in the reference tables incorporate practical considerations and safety factors, aiding identification of feasible designs. However, the solutions obtained are not only sub-optimal, but provide unclear information or insight needed to obtain an improved solution. An expert designer may be able to guide the design toward an improved design, but a novice designer may only be equipped to follow established design procedures, limiting design performance.

Stepping sequentially through PBD also results in accumulation of design data. Early estimates have significant effects on the final solution. Moving beyond design feasibility toward design improvement requires multiple iterations, which can be cumbersome. On the other hand, it is fairly easy to obtain a set of 'good enough' solutions to design variables which are suitable for practical use.

3.3 Hybrid Gear Design Strategies

We begin by analyzing PBD for spur gears. To transition to a hybrid strategy, we inspect steps that involve specifying a design variable value. Here designers must choose an initial specification for the number of pinion teeth. Shigley provides a reliable mathematical model for estimating a minimum N_p to avoid non-involute contact (interference) [11]:

$$N_p = \frac{2}{(1+2m_g)\sin^2\phi}\left(m_g + \sqrt{m_g^2 + (1+2*m_g)\sin^2\phi}\right) \quad (8)$$

Continuing from this more accurate initial estimate, the procedure requires approximation of two other quantities: center distance and face width. This constitutes the first step towards a hybrid design strategy H_1 for gear design.

An optimization subproblem is defined to determine face width, F. Diametral pitch, the only unspecified variable needed at this step for center distance approximation, is chosen from the standard diametral pitch values available in Ref. [13]. Thus, design rules pertaining to diametral pitch as specified in PBD are followed, and it is passed to the optimization algorithm as a fixed design parameter. Equation (7) and (8) from the previous section are chosen to be the stress constraints. Packaging constraints are added to limit the minimum and maximum gear sizes, eliminating degenerate or unbounded solutions. The objective function, as defined in Eqn.(4), is plotted over a range of pinion teeth N_p values. Optimal values of face width for minimizing $f(\mathbf{x})$ are obtained for a range of fixed P_d and N_p values. Bounds for F are estimated from the design rules pertaining to materials available and manufacturing methods. Figure 4 illustrates the results obtained using H_1.

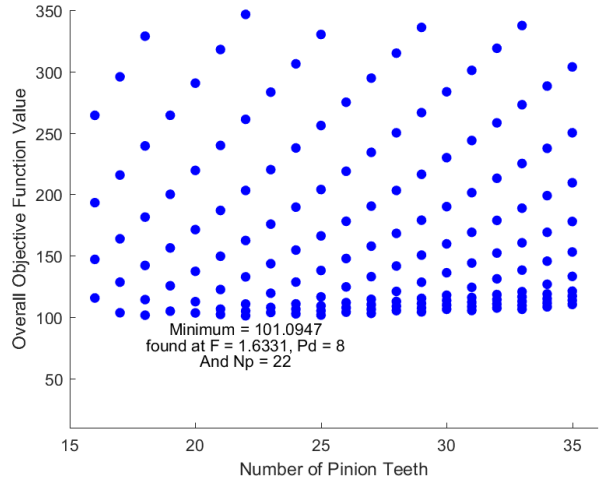

FIGURE 4. RESULTS FROM HYBRID STRATEGY 1, WHERE THE OPTIMIZATION SUBPROBLEM INCLUDES FACE WIDTH, F, AS AN OPTIMIZATION VARIABLE FOR FIXED VALUES OF DIAMETRAL PITCH, P_d, AND THE NUMBER OF PINION TEETH, N_p, IS OBTAINED VIA PBD.

Significant advantages of using the hybrid strategy can be observed immediately. Since an outer loop passed standard P_d and N_p values to the optimization subproblem, a clear optimal solution was obtained and plotted. While the overall objective function is minimized (compactness maximization), optimal face width values for a range of P_d values can be obtained. As mentioned in the previous section, the diametral pitch determines the tooling to be used to machine gears. If standard tooling is the only option available for design engineers, this solution set provides valuable insight. The solution strategy, compared against design rules and suggestions within the reference tables, instills valuable *inferential intuition* (defined in [25]).

Hybrid Design Strategy H_2 is guided by a similar approach, but with a different starting point. Another single design variable optimization subproblem is defined with P_d as the optimization variable. Face width is to the optimization subproblem from the outer loop as a fixed parameter.

Face width is one of the most important factors in gear manufacturing. Often face width may be restricted to standard sizes, or otherwise prescribed, driving the rest of the gear design as opposed to diametral pitch. Hybrid design strategy H_2 provides excellent insight to help make design decisions in such situations. The results obtained, shown in Fig. 5, are significantly better than those from PBD, and similar to results from H_1.

Hybrid Design Strategy H_3 involves combining strategies

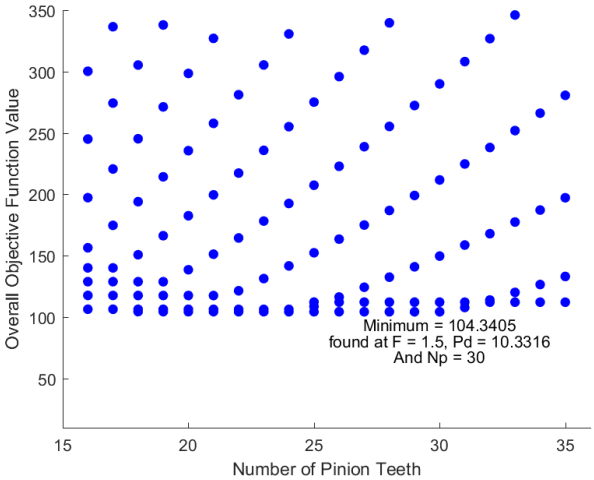

FIGURE 5. RESULTS FROM HYBRID STRATEGY 2, WHERE THE OPTIMIZATION SUBPROBLEM INCLUDES DIAMETRAL PITCH, P_d, AS AN OPTIMIZATION VARIABLE FOR FIXED VALUES OF FACE WIDTH, F, AND THE NUMBER OF PINION TEETH, N_p, IS OBTAINED VIA PBD.

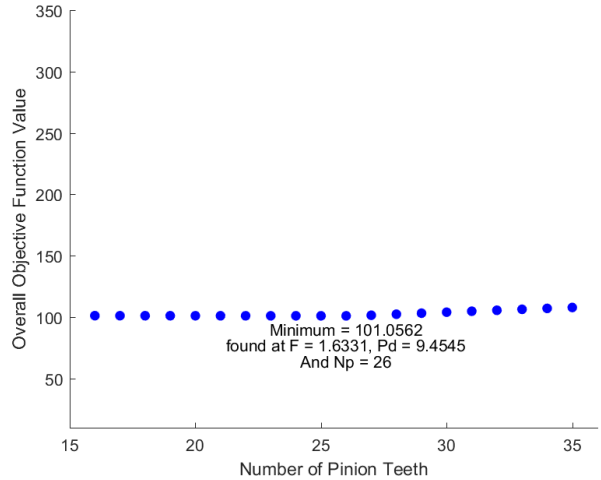

FIGURE 6. RESULTS FROM HYBRID STRATEGY 3, WHERE THE OPTIMIZATION SUBPROBLEM INCLUDES DIAMETRAL PITCH, P_d, AND FACE WIDTH, F, AS OPTIMIZATION VARIABLES FOR FIXED VALUES OF THE ALLOWABLE NUMBER OF PINION TEETH, N_p, OBTAINED VIA PBD.

H_1 and H_2. This strategy moves into the realm of solving a multi-variate constrained optimization problem. The strategy aims eliminates additional design rules, which may have been less obvious earlier, by adding constraints to the optimization problem. Practical considerations for bounds on the two design variables are entered as constraints. The value of the objective function is plotted over the third design variable in Fig. 6. This third variable is a fixed parameter in the optimization subproblem, passed in by the outer loop.

Preliminary results indicate that using hybrid solution strategies is promising. Here we enumerated different solution strategies by replacing design rules at different steps in PBD with optimization, and observed the resulting objective function values. With a relatively small increase in design effort, computational time for each iteration is reduced significantly, while design solutions quality is improved, steering efforts toward optimality. Being a modular approach, augmenting PBD with optimization can be carried out at any stage. Once it is implemented, it also allows easy modification of the objective function, constraints, and design variables.

3.4 Optimization-Based Gear Design

As a final comparison for hybrid methods, we implement a design solution strategy here where all decisions are made simultaneously using optimization. This requires development of models associated with constraints, enabling prediction of system behavior with respect to changes in all three design variables. This approach is articulated below.

Design Variables:

Face Width F
Number of Pinion Teeth N_p
Diametral Pitch P_d

Constraints:

Bending Stress and Contact Stress: As defined in Eqns. (6) and (7), suitable safety factors are incorporated into stress constraints to avoid tooth failure.
Contact Ratio: The contact ratio must be constrained to ensure gear teeth to not have non-involute contact (interference):

$$1.2 \leq m_c \leq 2.0$$
$$\text{where:} \quad m_c = \frac{\left(\sqrt{{d_{o,p}}^2 - {d_{b,p}}^2} + \sqrt{{d_{o,g}}^2 - {d_{o,p}}^2} - 2Csin(\phi)\right) P_d}{2\pi cos(\phi)} \tag{9}$$

and $d_{b,p}$ and $d_{b,p}$ are the base circle diameters of the pinion and gear, respectively, and C is the center distance.

Bounds on Face Width: The gear face width constraint is:

$$\frac{3\pi}{P_d} \leq F \leq \frac{5\pi}{P_d} \tag{10}$$

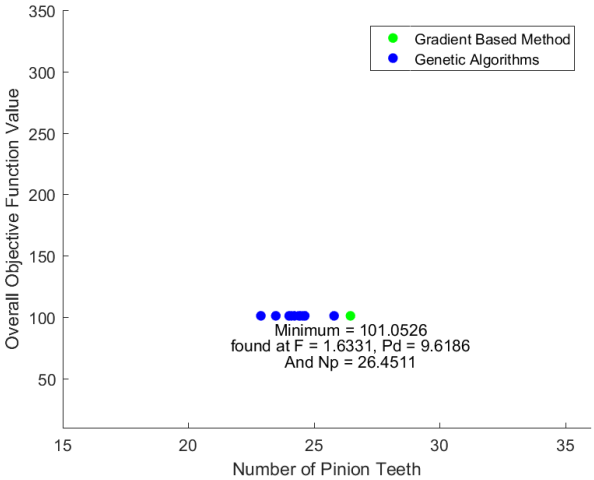

FIGURE 7. RESULTS USING AN OPTIMATION BASED APPROACH WHERE ALL DESIGN VARIABLES ARE INCLUDED AS OPTIMIZATION VARIABLES. GRADIENT BASED ALGORITHMS AND GENETIC ALGOIRTHMS PROVIDE RESULTS SIMILAR TO RESULTS OBTAINED THROUGH HYBRID STRATEGIES, INCREASING CONFIDENCE IN THE HYBRID SOLUTION STRATEGY.

Packaging Constraints: The minimum gear size is limited by shaft size, and the upper bound was chosen arbitrarily. Since the objective is to reduce size and weight, this upper bound does not influence the design.

The optimization problem was solved using both gradient-based and evolutionary optimization algorithms. Figure 7 plots the objective function values obtained, showing results with different numbers of pinion teeth. The optimization problem was well-constrained, and determined all three design variables, eliminating the need to use PBD. Design rules are considered within the constraint functions, but are limited to solution finding and not included in the design decision-making process.

The optimization strategy produces designs with variable values that are slightly different designs produced using the hybrid strategies, and improvement in objective function value is minimal. This shows that even the simplest hybrid strategy can help find near-optimal solutions, but at significantly reduced effort. We expect that as design problem complexity increases, shifting more design variables to optimization subproblems would be required to realize significant performance improvement.

4 Conclusions and Future Work

This article introduces the concept of augmenting existing procedure-based design strategies with optimization subproblems to take advantage of the unique complementary benefits of PBD and design optimization. Hybrid Design Strategies were conceptualized and compared to existing design strategies. An illustrative example based on gear design was used to create, implement, and analyze PBD, hybrid, and optimization-based solution strategies.

Initial evidence indicates that there may be some synergy between using PBD for some design decisions, and using optimization for others. When making solution strategy decisions, designers must balance between overall design effort and solution quality. Here we discovered that solution quality could be increased significantly with only modest development effort by determining some design variables using optimization. This work lays the foundation for ongoing work focused on developing a more general framework to assist designers in deciding how to construct a hybrid solution method that best suits their needs.

Future work will involve a deeper study of how to construct hybrid implementations effectively including principles to assist designers in making decisions regarding optimization subproblem formulation and Hybrid Design Strategies for their specific design problems. The goal of creating such principles and a more general framework is to aid the intuition of designers and to lead to improved quality of practical design solutions in the early stages of design with marginal increase in developmental design and modeling effort.

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