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EFFICIENT PARAMETERIZATION OF LARGE-SCALE DYNAMIC MODELS THROUGH THE USE OF ACTIVITY ANALYSIS

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ABSTRACT

Previous research presents both sensitivity-based and principal component-based techniques for improving the tractability of system identification. Both have proven viable, but the former may be computationally inefficient for large problems, and the latter require a change of realization that may compromise the physical meaning of the parameters to be identified. This paper proposes for the first time the use of activity analysis, an efficient and realization-preserving model reduction technique, for identification space reduction. Theoretical and numerical studies highlighting the viability of activity analysis versus the previous two methods are presented.

INTRODUCTION

Simulation models are commonly used for the design, validation, and control of dynamic engineering systems. Such models range from empirical to physics-based, the latter being particularly useful for gaining insight into system dynamics and developing new system designs and controls. One major challenge in using such physics-based first principles models is identifying their parameters, especially if the models are large and nonlinear. As the size and number of parameters of a dynamic model grows, the computational expense of identifying these parameters also grows, often exponentially [1]. One way to alleviate this computational burden is to determine which parameters in the model are most important to the system's dynamics and outputs, and then only identify those parameters. This has been shown in [2-9] to help improve the accuracy and efficiency of the identification problem for both linear and nonlinear models. The following section briefly reviews the techniques proposed in the literature for the selection of model parameters to identify, focusing on their strengths and weaknesses.

BACKGROUND

Two main classes of techniques have been used to select the most important parameters to identify in a model: sensitivity analysis, and principal component analysis (also know as proper orthogonal decomposition). This section discusses the strengths and weaknesses of each of these classes in the context of the problem at hand. Sensitivity methods focus on evaluating how errors in a model's parameters affect its outputs. An engineer can then focus on identifying those parameters to which the model output is most sensitive. There are several well-known methods for calculating the sensitivity of a model output with respect to model parameters for both linear and nonlinear systems. Alam and Sage [2] use a sensitivity-based technique to quantify *a priori* the loss in accuracy expected from eliminating parameters from an ARMA model. This method, however, is limited to linear ARMA models, and therefore too restrictive for the problem at hand. Benchulch and Chow [3] use DOE techniques to calculate a sensitivity matrix for each step in the optimal identification process for a nonlinear model. This method takes advantage of the change in parameters sensitivity as different system configurations are considered, but requires calculating a sensitivity matrix at each optimization step. This can be prohibitively computationally expensive for large models, as the costs of calculate this matrix rise rapidly with the size of the model. Velez-Reyes and Verghese [4] use another DOE-based technique for selecting an important subset of model parameters to identify. The results for this work show an improved efficiency with acceptable accuracy. However, this parameter selection process again can be computationally expensive. Additionally, the parameter subset selection is only done once, at the beginning of the identification problem. One should keep in mind, however, that as parameters change during an identification process, the set of important parameters may change as well. To summarize, the literature shows that sensitivity analysis is, in general, quite useful for selecting important parameters within the context of system identification. However, for the large models targeted by this work it is generally too computationally expensive.

Principal component analysis can be a viable alternative to sensitivity analysis as a tool for selecting the most important parameters to identify for a given model. Lee *et al.* [5] utilize this technique by taking snapshots of a system's state trajectory and computing the covariance matrix of these snapshots. The singular value decomposition of this matrix furnishes a transformation matrix that is then used to change the system model's realization. Using the singular value associated with each transformed state as a measure of its relative importance, Lee *et al.* truncate the least important states from the model. This reduces both the size of the model and the number of parameters to be identified. Moore [10] presents a similar technique, known as balanced truncation, to order states of a system from most important to least important, and then trim the unimportant states. This technique performs a basis change on the model such that each of the resulting state variables is equally observable and controllable. The states that are least controllable/observable can then be truncated from the model, not used for parameter identification, or residualized. This technique is commonly used because, if the model is linear, an upper bound on the error introduced through truncation can be calculated based on the Hankel singular values associated with each state [11]. Both of these techniques have been successfully used in the literature to reduce the number of parameters to be identified for a given model [5-9]. However, they are not realization-preserving, in the sense that they perform basis changes that often furnish new states and parameters for the model at hand. This may not be ideal, especially if the model is derived from first principles and its parameters are chosen to have appealing intuitive meanings.

In summary, the literature presents two classes of techniques for determining the most important parameters to identify for a given system model: sensitivity-based techniques and principal component-based techniques. Both have proven viable, but the former may not be computationally efficient for large system models, and the latter may compromise the physical meaning of parameters in a first principles model. This paper proposes a third technique that uses *activity* – an energy-based model reduction metric [12] – for selecting the most important parameters to identify. This metric calculates the relative importance of each state based on the total power flowing through that state during a time period of interest. By selecting only the parameters associated with the most active states for system identification, one may be able to significantly accelerate this system ID in an efficient, realization-preserving manner. This is validated here by comparing this proposed technique to both principal component analysis and sensitivity analysis through theoretical and numerical means.

PROBLEM STATEMENT

The goal of this work is to improve the efficiency of parameterizing large, nonlinear, first-principle models of dynamic systems. Current techniques within the literature can either be too computationally expensive, or defeat the purpose of using first-principles models. It is proposed that activity analysis can be used to estimate the sensitivity of a model's output to changes in the model's parameters more efficiently while preserving physical intuition gained from having a first-principles model. Theoretical and numerical examinations are done to examine the viability of this approach.

THEROETICAL ANALYSIS OF ACTIVITY

Activity was first introduced as a concept by Louca *et al.* [12]. It was developed as a tool for model reduction. Activity is defined as the total power flow through a dynamic element during a time period of interest and is calculated using the following equation:

$$Activity = \int_{T_1}^{T_2} \left|P(t)\right| dt \qquad (1)$$

The power flow in an energetic element throughout the simulation is represented by $P(t)$. As an example, for a spring this power flow is defined as the instantaneous velocity across the spring multiplied by the force through the spring. Integrating the absolute value of this power flow over time furnishes activity: a measure of the *total* power flow through the spring. This should not be confused with *net* energy change, which, for a unity-stiffness spring, is given by:

$$E_i = \left| \frac{x_i^2(T) - x_i^2(0)}{2} \right| \qquad (2)$$

It has been posited that those elements with the highest percentage of power flow (i.e., activity) have the most effect on system dynamics. Parameters of those elements may also be critically important. Since the activity associated with an element is not dependent on the number of inputs or outputs a model has, it can be easily applied to both SISO and MIMO systems. There is previous research [13,14] examining the link between activity and sensitivity of model parameters, but this previous work explores using a combination of the two approaches as a design methodology. It does not explore the possibility of using activity alone to estimate the sensitivity of model outputs to various parameters.

What follows is an outline of a mathematical connection between balanced truncation and activity analysis in specific cases. It is shown that, under certain conditions, balanced truncation and activity analysis are equivalent.

To show this mathematical connection, first, Hankel singular values are compared to the net change in energy of the states of a special case system. Then the net change in energy is compared to the activity of the energetic elements. By showing that, in a special case, the activity of the energetic elements is proportional to the Hankel singular value associated with that state, a link can be made between balanced truncation and activity analysis.

Consider a minimal, LTI, state-space representation of a discrete-time system of the form:

$$\begin{aligned} x(k+1) &= Ax(k)+Bu(k) \\ y(k) &= Cx(k)+Du(k) \end{aligned} \tag{3}$$

The observability and controllability grammians of this system are calculated by:

$$W_c = \sum_{k=0}^{\infty} A^{k^T} BB^T A^k \tag{4}$$

$$W_o = \sum_{k=0}^{\infty} A^{k^T} C^T CA^k \tag{5}$$

Moore [10] first showed that if one has a minimal LTI state-space representation of a discrete-time system one can find a similarity transformation to express the system as a balanced realization where each state is equally controllable and observable. This system will be in the form:

$$\begin{aligned} &\overline{x}(k) = Sx(k) \\ &\overline{A} = S^{-1}AS \\ &\overline{B} = S^{-1}B \\ &\overline{C} = CS \\ &\overline{D} = D \\ &\overline{W}_c = \overline{W}_o = diag(\sigma_1,\sigma_2,\ldots,\sigma_n) \\ &\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n \end{aligned} \tag{6}$$

The states of this minimal balanced realization can be ordered in such a way that the first states in the state vector have the most effect on system outputs, while the states at the end of the state vector have the least effect on system outputs. This is quantified by the Hankel singular values (σ_i), which rank the states in a quantitative way. Every state has an associated Hankel singular value, and those states with larger values have more effect on system dynamics and outputs than those with smaller values. Glover [11] extended this theory to prove that if a system is expressed as a balanced realization then that system is minimally sensitive to parameter errors and noise, and that errors in parameters associated with the larger Hankel singular values have more effect on the system output.

Further work in this area has drawn a connection between the Hankel singular values of a balanced realization and the activities of a model developed from said balanced realization. This work shows a fundamental concordance between balanced realization and activity [15] in the continuous-time domain. In [15], Fathy *et al.* prove that, for a special case of an LTI balanced realization of a system, it is possible to construct a bond graph model of the system such that the activities of the energetic elements of the bond graph are equal to one-half the Hankel singular values of the balanced realization.

By combining Fathy's work with Moore's, and Glover's, it is possible to prove that the output of a bond graph model derived based on the procedure posed by Fathy will be minimally sensitive to errors in its parameters. Furthermore, errors in those parameters associated with the highest activities, and therefore Hankel singular values, will have the most effect on the system output. This shows a theoretical link between activity and principal component analysis under certain conditions. Unlike balanced truncation, activity analysis is realization-preserving, making it a good choice as an estimator of parameter sensitivity for the problems under consideration.

To show this, let us consider the L_2 norm of the output of the system in (6):

$$L_{2out} = \sqrt{\sum_{k=0}^{\infty} y^T(k)y(k)} \tag{7}$$

If we assume the input is zero at all k, then this expression is equivalent to:

$$L_{2out} = \sqrt{\sum_{k=0}^{\infty} x^T(k)C^T Cx(k)} \tag{8}$$

However, since there is no input, the states at time $k \geq 0$ can be expressed as the initial condition $x(0)$ multiplied by A^k. This leads to the following substitutions into eq. (8):

$$x(k) = A^k x(0) \tag{9}$$

$$L_{2out} = \sqrt{x^T(0)\left(\sum_{k=0}^{\infty} A^{k^T} C^T CA^k\right)x(0)} \tag{10}$$

However, we can substitute from (5) the observability grammians to get:

$$L_{2out} = \sqrt{x^T(0)W_o x(0)} \tag{11}$$

Let us construct a set of initial conditions with $x_i(0)=1$, and $x_j(0)=0$ for all $j \neq i$ If we choose one such initial condition and calculate the L_2 norm associated with that condition we get:

$$L_{2out,i} = \sqrt{x_i(0)\sigma_i x_i(0)} = \sqrt{\sigma_i} \tag{12}$$

Without loss of generality, let us assume that we can construct a bond-graph with unity inductances and capacitances to represent the system in (6). Fathy [15] has shown that this is possible for any minimal balanced realization of a system. The instantaneous energy in each state is then:

$$P_i(k) = \frac{x_i^2(k)}{2} \tag{13}$$

The activity associated with each energetic element over a time period of interest becomes:

$$A_i = \sum_{k=0}^{T-1} \frac{\left|x_i^2(k) - x_i^2(k+1)\right|}{2} \tag{14}$$

Now, assuming that none of the state trajectories change sign, then the absolute value and summation operators can be switched to give us:

$$A_i = \left|\sum_{k=0}^{T-1} \frac{x_i^2(k) - x_i^2(k+1)}{2}\right| \tag{15}$$

Which simplifies to:

$$A_i = \left|\frac{x_i^2(0) - x_i^2(T)}{2}\right| \tag{16}$$

The activity in each element is then also equal to the net energy change in a state defined in equation (2). Let us now construct an initial condition using our results from (11) as follows:

$$x_i(0) = L_{2out,i} = \sqrt{\sigma_i} \tag{17}$$

This gives an initial condition which is a vector of the roots of the Hankel singular values. Let us simulate the model from zero until infinity, and calculate the activity associated with each element.

$$A_i = \left|\frac{x_i^2(0) - x_i^2(T)}{2}\right| = \frac{x_i^2(0)}{2} = \frac{L_{2out,i}{}^2}{2} = \frac{\sigma_i}{2} \tag{18}$$

This shows that for a specific case of a discrete, LTI system the activity associated with each state will be equal to half the associated Hankel singular value. This result is analogous to the continuous time result shown by Fathy [15]. However, Glover [11] has shown that the system output is most sensitive to parameters associated with larger singular values. Since, for this case, the activity is linearly proportional to the singular values, the output is also most sensitive to changes in parameters associated with the larger activities. This suggests a mathematical link between activity analysis and balanced truncation in special cases. Next, a numerical example showing the relationship between activity analysis and sensitivity analysis is presented.

A case study involving a simple dynamic model of a vehicle will be shown to evaluate the viability of activity as an estimator of parameter sensitivity. A technique called AVASIM will be used to help evaluate the accuracy of a tuned model. AVASIM [16-18] is the "Accuracy and Validity Algorithm for SIMulation," and generates a Performance Index based on tolerances chose by the user. As used here this method is similar to scaled residual sums, with the caveat that a Performance Index of 1 corresponds to 100% accuracy; a Performance Index of 0 corresponds to the limit of our tolerance (in this case 99% accuracy); and a Performance Index of less than 0 indicates the model is invalid with respect to the tolerances chosen.

CASE STUDY

Model Description

This case study demonstrates that activity analysis and sensitivity analysis provide similar information for a specific system. The system used for this case study is the common quarter-car model (Figure 1). This model has six parameters; the sprung and unsprung masses (*Ms* and *Mus*), the tire stiffness and damping coefficients (*Kt* and *Bt*), and the suspension stiffness and damping (*Ks* and *Bs*). This simple model of a vehicle can be used to estimate the first two natural frequencies of a vehicle in the vertical direction.

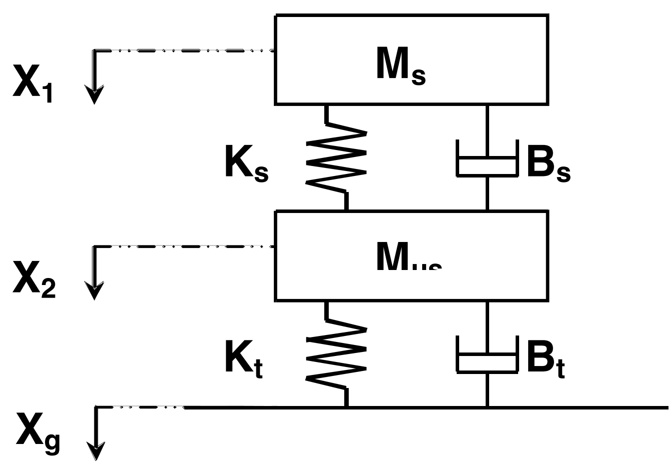

FIGURE 1: QUARTER CAR MODEL

Parameters for this model are used to represent a generic passenger vehicle. Two different sets of input/output pairs are used to examine the proposed procedure. The first input to the model is a step input displacement at the ground. The first output is the net force on the unsprung mass. It is very common for the tire damping coefficient to be dropped from this model, but it is included here to analyze the identifiability of that parameter. The model is first simulated at the nominal parameter values to provide the benchmark data to which other data sets will be compared.

First Experiment: Displacement Input

To provide a best-guess initial condition, the parameters are perturbed from their nominal values by random amounts up to 30%. Figure 2 shows the outputs of the system with both the nominal and initial condition set of parameters.

A Simulink [19] model was developed of this system. The “fmincon” function in Matlab [20] was used to attempt to tune the model parameters back to their original values. This was done by using AVASIM to compare the output from the perturbed model to the output from the nominal model. The goal was to maximize the objective function with respect to the model parameters. The optimization procedure converged to the nominal point. This took 376 function evaluations.

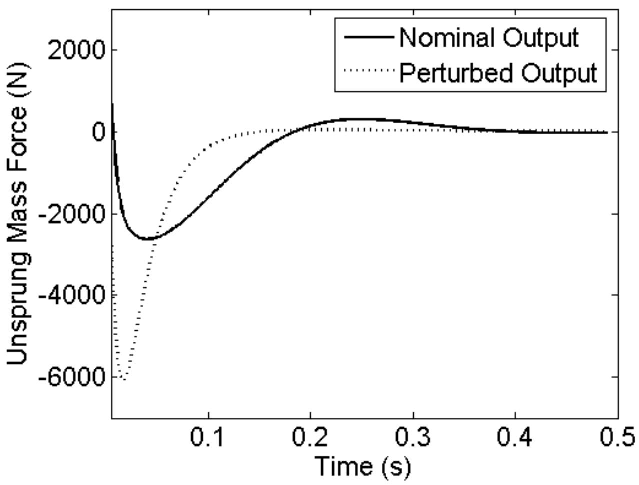

FIGURE 2: NOMINAL VS. INITIAL OUTPUT, DISPLACEMENT INPUT

Activity and Sensitivity Analysis

Both activity and sensitivity analyses of the results were done in order to select the important parameters of this model. First the activity of the model elements at the nominal was calculated (Figure 3). From this analysis it appears that the suspension stiffness and tire damping are associated with inactive elements.

FIGURE 3: ACTIVITY OF PARAMETERS

To confirm this fact a full factorial ANOVA experiment was performed around the nominal point to calculate the sensitivity of the model output to changes in the parameters (Table 1). The results of this analysis agree very strongly with the results from the activity analysis, i.e. the output is insensitive to changes in the suspension stiffness and tire damping.

TABLE 1: QC MODEL PARAMETER SENSITIVITIES, 1st EXPERIMENT

Source	Sum Sq.	d.f.	Mean Sq.	F	Prob>F
Ms	0.00639	1	0.00639	12.83	0.0009
Mus	0.02388	1	0.02388	47.97	0
Ks	0.00002	1	0.00002	0.04	0.8486
Kt	0.00734	1	0.00734	14.74	0.0004
Bs	0.00001	1	0.00001	0.02	0.8957
Bt	0.00027	1	0.00027	0.55	0.4635
Ms*Mus	0.16003	1	0.16003	321.41	0
Ms*Ks	0.00202	1	0.00202	4.06	0.0502
Ms*Kt	0.04418	1	0.04418	88.73	0
Ms*Bs	0.06391	1	0.06391	128.36	0
Ms*Bt	0.00065	1	0.00065	1.3	0.2608
Mus*Ks	0.00185	1	0.00185	3.71	0.061
Mus*Kt	0.2041	1	0.2041	409.91	0
Mus*Bs	0.00319	1	0.00319	6.41	0.0151
Mus*Bt	0.00493	1	0.00493	9.9	0.003
Ks*Kt	0.00156	1	0.00156	3.13	0.084
Ks*Bs	0.00044	1	0.00044	0.88	0.3532
Ks*Bt	0.00001	1	0.00001	0.02	0.8756
Kt*Bs	1.41676	1	1.41676	2845.44	0
Kt*Bt	0.00586	1	0.00586	11.78	0.0014
Bs*Bt	0.00022	1	0.00022	0.45	0.5053
Error	0.02091	42	0.0005		
Total	1.96854	63			

As an example of the usefulness of only tuning sensitive parameters, the optimization was carried out again with the two unimportant parameters, *Ks* and *Bt*, held constant. The results were a small loss in accuracy (PI of 0.919), with a twofold increase in efficiency (197 function evaluations), with the other 4 parameters achieving their nominal values.

Second Experiment: Harmonic Force Input

A second experiment was done to explore how the activity and sensitivity measures change for different inputs and outputs. For the second set of input/outputs, the input is a harmonic force in the vertical direction applied to the sprung mass. The output is chosen to be the displacement of the unsprung mass. The same model developed for the last experiment was modified for the new input and output, and the same procedure was followed. Once again the optimization routine tuned the model parameters to the nominal values. This time 229 function evaluations were required.

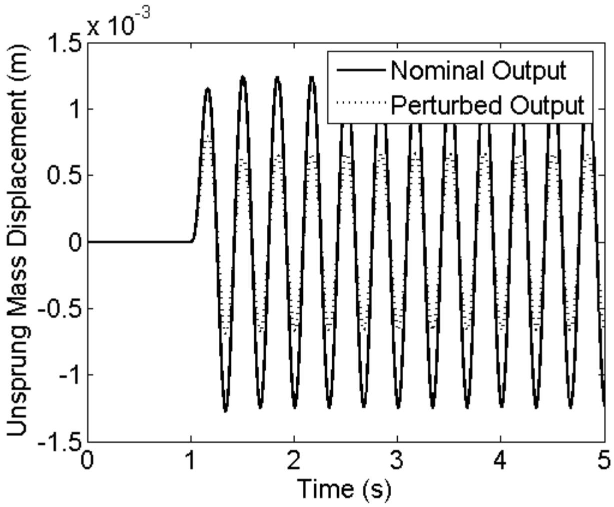

FIGURE 4: NOMINAL VS. INITIAL OUTPUT, FORCE INPUT

Once again the activity and sensitivity data at the nominal was calculated. In the activity information (Figure 5) for this experiment, the same parameters as before are deemed inactive, but the unsprung mass is also deemed inactive. This implies that for this choice of input/output the unsprung mass has become unimportant to the model output. Analysis of the sensitivity information in Table 2 agrees with this analysis.

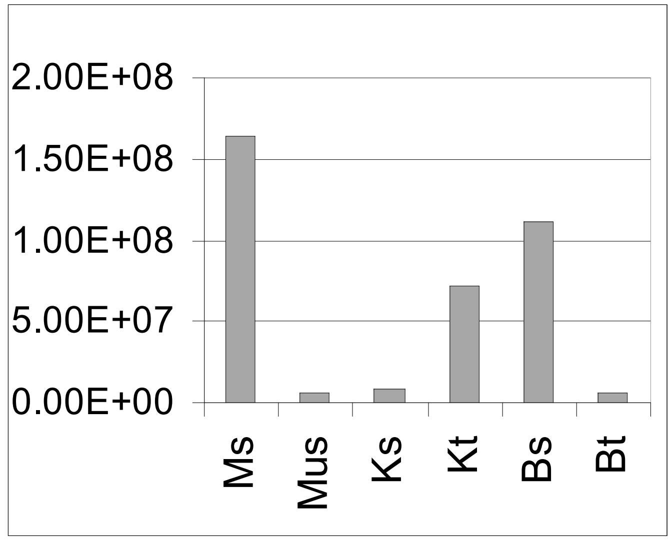

FIGURE 5: ACTIVITY OF PARAMETERS

TABLE 2: QC MODEL PARAMETER SENSITIVITIES, 2ND EXPERIMENT

Source	Sum Sq.	d.f.	Mean Sq.	F	Prob>F
Ms	0.00312	1	0.00312	334.72	0
Mus	0.00005	1	0.00005	4.9	0.0323
Ks	0	1	0	0.12	0.7273
Kt	0.0021	1	0.0021	225.05	0
Bs	0.00418	1	0.00418	448.21	0
Bt	0.00002	1	0.00002	1.78	0.1888
Ms*Mus	0.00001	1	0.00001	1.4	0.2433
Ms*Ks	0.00059	1	0.00059	63.58	0
Ms*Kt	0.0007	1	0.0007	74.64	0
Ms*Bs	0.15613	1	0.15613	16752.75	0
Ms*Bt	0.00067	1	0.00067	72.25	0
Mus*Ks	0	1	0	0.52	0.4735
Mus*Kt	0.00331	1	0.00331	355.32	0
Mus*Bs	0.00009	1	0.00009	9.86	0.0031
Mus*Bt	0	1	0	0.01	0.9148
Ks*Kt	0.00203	1	0.00203	218.09	0
Ks*Bs	0	1	0	0.1	0.7563
Ks*Bt	0	1	0	0.12	0.7306
Kt*Bs	0.06255	1	0.06255	6711.22	0
Kt*Bt	0	1	0	0.17	0.6851
Bs*Bt	0.00007	1	0.00007	7.41	0.0094
Error	0.00039	42	0.00001		
Total	0.23602	63			

Again the optimization was carried out a second time with the three unimportant parameters, *Mus*, *Ks*, and *Bt*, held constant. The results were no loss in accuracy (PI of 1.000), with a threefold increase in efficiency (69 function evaluations), with the other 3 parameters achieving their nominal values. These two numerical experiments highlight that activity analysis and sensitivity analysis provide much the same information for this simple system.

DISCUSSION

Based on the results, several points can be made. First, the similarity of the results from the activity and sensitivity analyses strongly indicate that activity can be used to estimate parameter sensitivity. The main advantage to this is that sensitivity analysis in this case took 64 simulation runs in both cases (this can be reduced, but must always be at least equal to the number of parameters), while activity analysis by its nature always takes just one. Also, the run required for activity analysis can be the same one used to calculate the objective function in an optimization procedure. This means that for models that have significant computational expense the activity information can be gathered with almost no additional cost, while the sensitivity information may take dozens of runs to compute. This shows how activity analysis can be as efficient as principal component analysis.

Second, if activity analysis is done prior to parameter identification, it may indicate which parameters within a model are not very identifiable for a given input and output. This might allow for the better design of experiments to ensure that parameters which are very important for the use of the model can be identified from the inputs and outputs chosen. Since activity analysis does not require a change of basis, we preserve our understanding of the physical meaning of the parameters in the same way as sensitivity analysis.

Finally, since activity information is so easy to get it may be possible to develop an optimization procedure that continuously updates the set of parameters being identified. This means that if a parameter becomes unimportant based on the activity analysis it will be dropped from the identification parameter set. Conversely, if a parameter becomes important it can be added to the identification set. By updating the set of parameters as the optimization is carried out one can insure that those parameters, and only those parameters, which are important to the model output are identified.

As a caveat one should be aware that all of the methods described in this work, with the exception of balanced truncation, are dependent on the input to the model having sufficient richness to excite all the important modes in the physical system. This is a separate problem that is much studied in the literature, but addressing it here goes beyond the scope of this work. It is assumed in this work that a properly rich input can be found and applied to the system in question.

It should also be noted that the results shown here are probably not sufficient to fully confirm the hypothesis that activity can be used as an estimator of model sensitivity. A validation of these results in a real world case study should be carried out. Applying this procedure to a real system may have some challenges in that a model may not contain dynamics present in the real system. This may require filtering of the data or careful selection of inputs. Furthermore the stochastic nature of real systems may require some adjustments to the proposed optimization procedure. However, this work indicates that this further research is likely to agree with the results shown here.

CONCLUSION

In this work a theoretical examination of the links between activity and principal component analysis was made. Furthermore, a simple model was used to analyze the viability of using activity as an estimator of parameter sensitivity for identification problems. This method has the potential to increase the efficiency of large-scale model identification problems like principal component analysis, while preserving our physical understanding of what the parameters mean like sensitivity analysis. It has been shown that there is a strong correlation between activity, sensitivity, and principal component analysis, though further examination and evaluation is warranted.

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APPENDIX

TABLE 3: NOMINAL PARAMETER VALUES

Ms	**2000 kilograms**
Mus	**200 kilograms**
Bs	**35000 Ns/m**
Bt	**1760 Ns/m**
Ks	**79000 N/m**
Kt	**790000 N/m**