1G.3: Rheological Design for Efficient Fluid Power

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Motivation

- Decrease friction in lubricated sliding contact
  - Decrease shear stress
  - Increase normal force
  - Approach: surface textures and Non-Newtonian fluids
- Determine optimal design of surface textures and lubricant
Motivation (Continued)

Weissenberg

\[ Wi = \lambda \dot{\gamma} \]

Non-linear: Amplitude dependent properties

Viscoelastic: Time-dependent properties

Deborah

\[ De = \frac{l}{t_{char}} \]

Motivation (Continued)

Pipkin space [1]

Controlled by geometry

Geometry + Fluid

Motivation (Continued)

\[ Wi = \lambda \dot{\gamma} \]

- Generalized Newtonian Fluid
- Ordered Fluid Expansion
- Non-linear models

\[ De = \frac{l}{t_{char}} \]

Linear Viscoelastic
Outline

- Experimental precision and challenges
- Experimental results surface textures and Newtonian fluids
  - Origin of the Pipkin space
- Experimental results surface textures and Non-Newtonian fluids
  - Exploring more of Pipkin space
Materials Tested
Non-dimensional ratios govern behavior [2]

Materials Tested: Newtonian Lubricants

![Graph showing the viscosity (η [Pa s]) of two Newtonian lubricants (S600 and S60) as a function of temperature (°C). The graph illustrates the decrease in viscosity with increasing temperature.]
Experimental Setup

- Gap controlled rotational rheometer
  - DHR-3 by TA instruments
  - Precision aligned for tribo-rheometry [2]
- Parallel disks D=40 mm
  - Top: flat ($R_{RMS}=3.33 \, \mu m$), rotating, stainless
  - Bottom: textured, 1018 steel, attached with Crystalbond

Key Challenges:
- Gap error
- Non-texture normal forces

Gap Error

- Risk of misinterpreting shear stress reduction that is not due to the textures [3,4]
- Gap zeroing calibration based on contact force
- Squeeze flow of air produces force
- Calibrated $\varepsilon = 19.0 \pm 0.69$ μm using Newtonian oil with $\eta = 0.14$ Pa s

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\[
a = \frac{t}{1+\frac{1}{h_a}}
\]

\[
\frac{1}{a} = \frac{1}{t} + \frac{1}{h_a}
\]

---

Non-Texture Normal Forces

- Risk of misinterpreting normal forces that are not due to the surface textures [5-7]
  a) Inertia: \( F_{\text{inertia}} = \frac{3}{40} R^4 \) \( h^2 \)
  b) Surface Tension: \( F_N = C \)
  c) Non-Parallelism: \( F_{np} = \frac{R^4}{h^2} \left( 0.256 \frac{h}{h} \right) \)

Shear Stress Reduction: Newtonian Fluids

Real shear stress reduction through use of textures

\[
a = \frac{2(h_a + e)}{R^4} M
\]
Asymmetric textures produce forces above experimental limit through viscous effects.
Effective Friction Coefficient

Asymmetric textures decrease friction. Optimal $\beta$. 

\[ \frac{F_t}{F_N} = \frac{M}{R} \]
Conclusions: Newtonian Fluids

- Surface textures decrease shear stress
- Symmetry must be broken in order to produce normal forces above experimental limit
  - Sign of force depends on direction of motion
- Normal forces are produced by viscous effects up to Re_h=1.21
- Optimal angle $\beta$ exists for decreasing friction with asymmetric surface textures
Polyisobutylene (PIB) has been used as an additive for enhancing mechanical properties [8] and modifying viscosity [9]

decrease temperature dependence of viscosity

- Dissolves in mineral oil
- 0.5wt% PIB (M_W~1,000,000) in mineral oil (highly refined, S6, η=9.62 mPa s)
- c/c*=0.0774 (dilute solution)

Non-Newtonian Rheology Characterization

\[ \eta = \eta_\infty + \frac{\eta_0 - \eta_\infty}{\left(1 + (\lambda \dot{\gamma})^2\right)^{(1-n)/\alpha}} \]

- \( \eta_\infty = 9.62 \text{ mPa s} \)
- \( \eta_0 = 30 \text{ mPa s} \)
- \( \lambda = 7.5 \text{ ms} \)
- \( n = 0.8125 \)
- \( \alpha = 2 \)
Shear Stress Reduction

\[ \eta_a = \frac{2(h_a + \varepsilon)}{\pi R^4} \left( \frac{3}{4} + \frac{1}{4} \frac{d \ln(M)}{d \ln(\dot{\gamma}_R)} \right) \frac{M}{\Omega} \]

Surface textures reduce viscosity beyond shear thinning
Normal Force Production

Asymmetric surface textures produce normal forces above viscoelastic response.
Effective Friction Coefficient

Asymmetric textures decrease friction. Optimal $\beta$
Conclusions: Non-Newtonian Fluids

- Surface textures decrease shear stress beyond shear thinning.
- Symmetry must be broken to produce normal forces above viscoelastic response:
  - Normal forces are always positive.
- Optimal angle $\beta$ exists for decreasing friction with asymmetric surface textures:
  - Friction coefficient is smaller with Non-Newtonian fluids than Newtonian.
Future Work

- Examine relaxation time scale effects
  - Change concentration of polymer in solution
  - Explore more of Pipkin space

- Mathematically model surface textures and Non-Newtonian fluids
  - 2\textsuperscript{nd} order fluid
  - 3D flow theorem of Giesekus with Reynolds equation solver

- Determine optimal design of textures and fluid
  - Direct optimization with Reynolds equation
  - Adaptive surrogate modeling techniques [10]
  - Experimentally test optimal texture and fluid

Acknowledgements and Contact Information

Ewoldt Research Group

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Non-Dimensionalization

\[
(F_N, a) = f(h, R, R_c, R_t, D, h, R, R_c, R_t, h, R, R_c, R_t, h, D)
\]

By Buckingham Pi Theorem:

\[
\left( \frac{F_N}{R^2 \left( \frac{R}{h} \right)^a} \right) = \left( \frac{h}{R}, \frac{h}{R_c}, \frac{h}{R_t}, \frac{h}{D}, h^2 \right)
\]
### Table

<table>
<thead>
<tr>
<th></th>
<th>$h$ [mm]</th>
<th>$D$ [mm]</th>
<th>$R_c$ [mm]</th>
<th>$R_t$ [mm]</th>
<th>$R$ [mm]</th>
<th>$\phi$ [rad]</th>
<th>$h/D$ [-]</th>
<th>$h/R_c$ [-]</th>
<th>$h/R_t$ [-]</th>
<th>$h/R$ [-]</th>
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</thead>
<tbody>
<tr>
<td>T1</td>
<td>0.069</td>
<td>0.513</td>
<td>3.655</td>
<td>0.770</td>
<td>5.130</td>
<td>0.628</td>
<td>0.135</td>
<td>0.019</td>
<td>0.090</td>
<td>0.013</td>
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<tr>
<td>T2</td>
<td>0.169</td>
<td>1.257</td>
<td>8.952</td>
<td>1.885</td>
<td>12.57</td>
<td>0.628</td>
<td>0.135</td>
<td>0.019</td>
<td>0.090</td>
<td>0.013</td>
</tr>
<tr>
<td>T3</td>
<td>0.269</td>
<td>2.000</td>
<td>14.25</td>
<td>3.000</td>
<td>20.00</td>
<td>0.628</td>
<td>0.135</td>
<td>0.019</td>
<td>0.090</td>
<td>0.013</td>
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<tr>
<td>T4</td>
<td>0.319</td>
<td>2.372</td>
<td>16.90</td>
<td>3.558</td>
<td>23.72</td>
<td>0.628</td>
<td>0.135</td>
<td>0.019</td>
<td>0.090</td>
<td>0.013</td>
</tr>
<tr>
<td>T5</td>
<td>0.419</td>
<td>3.115</td>
<td>22.20</td>
<td>4.673</td>
<td>31.15</td>
<td>0.628</td>
<td>0.135</td>
<td>0.019</td>
<td>0.090</td>
<td>0.013</td>
</tr>
</tbody>
</table>

### Equations

- $h = 1.4 \text{ Pa s}$
- $r = 846.4 \text{ kg/m}^3$

### Diagrams

- Left: Graph showing $\tau$ [Pa] vs. $\Omega$ [rad/s]
- Right: Graph showing $\eta$ [-] vs. $Re_n = \rho \Omega h^2 / \eta$ [-]
\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline
 & \( h \) [mm] & \( D \) [mm] & \( R_c \) [mm] & \( R_t \) [mm] & \( R \) [mm] & \( \beta \) [rad] & \( \varphi \) [rad] & \( h/D \) [--] & \( h/R_c \) [--] & \( h/R_t \) [--] & \( h/R \) [--] \\
\hline
T1 & 0.069 & 0.142 & 3.655 & 0.770 & 5.130 & 0.093 & 0.628 & 0.485 & 0.019 & 0.090 & 0.013 \\
T2 & 0.169 & 0.348 & 8.952 & 1.885 & 12.57 & 0.093 & 0.628 & 0.485 & 0.019 & 0.090 & 0.013 \\
T3 & 0.269 & 0.554 & 14.25 & 3.000 & 20.00 & 0.093 & 0.628 & 0.485 & 0.019 & 0.090 & 0.013 \\
T4 & 0.319 & 0.657 & 16.90 & 3.558 & 23.72 & 0.093 & 0.628 & 0.485 & 0.019 & 0.090 & 0.013 \\
T5 & 0.419 & 0.863 & 22.20 & 4.673 & 31.15 & 0.093 & 0.628 & 0.485 & 0.019 & 0.090 & 0.013 \\
\hline
\end{tabular}
\end{table}

\begin{align*}
\text{h} &= 1.4 \text{ Pa s} \\
\text{r} &= 846.4 \text{ kg/m}^3
\end{align*}

\begin{figure}
\centering
\begin{subfigure}{0.49\textwidth}
\centering
\begin{logloggraph}
\addplot[black, mark=square, mark options=square filled] coordinates { (0.1, 100) (1, 10) (10, 1) (100, 0.1) (1000, 0.01) };
\addplot[red, mark=triangle, mark options=triangle filled] coordinates { (0.1, 100) (1, 10) (10, 1) (100, 0.1) (1000, 0.01) };
\addplot[blue, mark=diamond, mark options=diamond filled] coordinates { (0.1, 100) (1, 10) (10, 1) (100, 0.1) (1000, 0.01) };
\addplot[green, mark=x, mark options=x filled] coordinates { (0.1, 100) (1, 10) (10, 1) (100, 0.1) (1000, 0.01) };
\addplot[orange, mark=star, mark options=star filled] coordinates { (0.1, 100) (1, 10) (10, 1) (100, 0.1) (1000, 0.01) };
\end{logloggraph}
\caption{\( \tau, p \) [Pa]}
\end{subfigure}
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\centering
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\addplot[black, mark=square, mark options=square filled] coordinates { (0.1, 100) (1, 10) (10, 1) (100, 0.1) (1000, 0.01) };
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\end{logloggraph}
\caption{\( \eta$, $F_N$ \)}
\end{subfigure}
\end{figure}

\begin{align*}
\eta &= 1.4 \text{ Pa s} \\
\text{r} &= 846.4 \text{ kg/m}^3
\end{align*}
S60 Shear Stress Reduction

![Graph showing shear stress reduction with edge shear rate and different plate angles.](image-url)
S60 Normal Force Production

![Diagram showing normal force production vs angular velocity for different plate geometries and thicknesses.](image-url)

- **A** (Flat plate, $\beta=5.3^\circ$)
- **B** (519 $\mu$m, $\beta=9.4^\circ$)
- **C** (269 $\mu$m, $\beta=14^\circ$)
- **D** (1019 $\mu$m, $\beta=21.7^\circ$)
- **E** (Symmetric)

**Legend:**
- Orange: 1019 $\mu$m
- Green: 519 $\mu$m
- Blue: 269 $\mu$m
- Gray: Exp Limit

**Axis:**
- $F_c - F_{c0}$ [N]
- Angular Velocity $\Omega$ [rad/s]
S60 Normal Force Scaling

![Graph showing force scaling with angular velocity for different plate geometries and thicknesses.](image-url)
S60 Effective Friction Coefficient

\[ \frac{F_T}{F_N} = \frac{M}{R} \]

![Graph showing effective friction coefficient for different angles and plate types.](Image)
Reynolds Equation Solver: Validation

![Graph showing the error in the computed solution as a function of the polynomial degree. The error is expressed as \( |F_{\text{true}} - F_{\text{comp}}| \) on a logarithmic scale, and the polynomial degree is on the x-axis. The graph includes a line with the label \( e^{-N} \) scaling and a note that \( R_o - R_i = 1e-6 \).]
Reynolds Equation Region of Applicability

- Left graph: Angular Velocity $\Omega$ vs. Total Gap Height $h$ [mm]
  - $h < 0.1R$
  - $Re_h < 0.1$
  - $Na < 0.1$
- Right graph: Asymmetric Texture Angle $\beta$ vs. Nominal Gap Height $h_0$ [mm]
  - $h_0 + D < 0.1R$
  - Applicability Region
Shear Stress Reduction: Reynolds Equation

\[ M = N_{tex} \int_{R_i}^{R_o} \int_{-j/2}^{j/2} z \, dr \, (rd) \, r \]

\[ a = \frac{2(h_a + \theta)}{R^4} M \]

![Graph showing data points and line indicating Reynolds vs Experimental shear stress reduction.](image-url)
Normal Force Production: Reynolds Equation

\[ F_{N_{\text{Reynolds}}} = F_{\text{exp}} \]

\[ F_{N_{\text{Experimental}}} \]

\[ |F_{N_{\text{Reynolds}}}| \]

\[ |F_{N_{\text{Experimental}}}| \]

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Effective Friction Coefficient: Reynolds Equation

\[ \mu^* \text{ vs. } \beta \text{ [°]} \]

- Exp
- Sim

- 1019 µm
- 519 µm
- 269 µm