Topology Optimization Formulations for Circuit Board Heat Spreader Design

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In this work, we investigate the effectiveness of several problem formulations for topology optimization of electronics (printed circuit board) thermal ground planes with emphasis on overall system efficiency. Many relevant existing studies have concentrated on heat extraction problems involving a homogeneously heated design domain. This is a helpful approximation that leads to efficient solutions, but manufactured circuit boards have several discrete heat sources, often spanning a wide range of power levels. Considering these discrete heat sources and aiming to optimize electrical system performance through topology optimization of passive heat spreaders introduces new challenges not addressed currently by established methods. A variety of new problem formulations, motivated by existing power electronic system design problems, are investigated here, including non-compliance objective functions, temperature constraints and targets, and electrical system efficiency calculations. Studies presented here focus on topology optimization of passive heat spreaders, although the methodology may be extended logically to systems involving active cooling. The effectiveness of traditional topology optimization techniques to handle changes in problem structure is assessed.

Introduction

The research presented here focuses on heat spreader (i.e., thermal ground plane) topology optimization for printed circuit boards (PCBs). PCBs have interesting thermal requirements as thermal behavior is coupled with the electrical performance. Changes in PCB temperature distribution impacts trace wire resistivity and internal resistance of PCB components, and component and trace behavior influences temperature distribution. This multidisciplinary design problem involves both strategic temperature reduction and design for electrical performance. Satisfying thermal and electrical performance requirements is challenging for several reasons, particularly when PCB component placement is determined *a priori* (often resulting in asymmetric layouts). Different strategies are explored here to capture electrical performance in the context of thermal system design where discrete heat sources are present. To address the design of heat spreaders, topology optimization techniques are demonstrated using several case studies.

Topology optimization traditionally revolves around the use of a compliance framework. An important review of topology optimization by Sigmund and Maute[1] identifies strengths and weaknesses of existing theory and methods for topology optimization. One limitation is the widespread use of objective functions that are analogous to structural compliance metrics. Topology optimization can be used for other problem formulations; investigations of these formulations, however, is limited. This article addresses new formulations motivated by realistic PCB problems. A limited number of researchers have explored unique problem formulations. Zhuang et al.[2] solved a heat conduction problem considering multiple discrete loads minimizing temperature gradients. There are several objective function explored by Dede et al. for printed

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circuit boards including: 1) Minimizing local temperature averages, 2) Minimizing thermal resistance, and 3) Minimizing average temperature.[3–5] Matsumori et al.[6] solved a thermo-fluid design problem under constant input power using two unique objectives: 1) Minimizing mean temperature on the domain, and 2) Maximizing heat generation on the domain. In this work, we explore different temperature-based problem formulations for heat conduction topology optimization exclusively.

I. Methodology

To explore different formulations for passive heat spreader design, SIMP-based topology optimization is used. The SIMP method considers a design domain of discretized elements where the material properties of each element are controlled by a material density parameter, ρ. A penalty function is included for each element to favor material distributions of 0 (void) and 1 (solid) on the design domain. Since this method considers each individual finite element as a design variable, the problem size is large. However, methods exist to calculate gradients efficiently, rendering this problem tractable. In this paper, a density filter is used to enforce minimum radius constraints. The optimization gradients are calculated using the adjoint method exclusively. Consider a general objective function, π, which may be in terms of temperature, T, heat flux, q, and material density, ρ:

$$\Theta = \int_{\Omega} \pi[T(\rho), q(\rho), \rho]. \tag{1}$$

In all of the following studies, the Dirichlet boundary will be fixed at zero, this results in the following simplification:

$$\Theta = \int_{\Omega} \pi[T^f(\rho), q^f(\rho), \rho], \tag{2}$$

where the objective function is not solely in terms of the unconstrained degrees-of-freedom. A "discretize then optimize" approach is used to obtain the sensitivities in terms of known quantities. The simplified objective is differentiated to obtain the derivative function. Since the residual of the finite element problem is equal to zero, the derivative of the residual equation is added to the derivative equation with a pair of Lagrange multipliers, λ_f and λ_p:

$$d\Theta = \frac{\partial \pi}{\partial \rho} + \frac{\partial \pi}{\partial T^f}\frac{\partial T^f}{\partial \rho} + \frac{\partial \pi}{\partial q^f}\frac{\partial q^f}{\partial \rho} + \lambda_f^T\left[\frac{\partial k^{ff}}{\partial \rho}T^f + k^{ff}\frac{\partial T^f}{\partial \rho} - \frac{\partial q^f}{\partial \rho}\right] + \lambda_p^T\left[\frac{\partial k^{pf}}{\partial \rho}T^f + k^{pf}\frac{\partial T^f}{\partial \rho} - \frac{\partial q^p}{\partial \rho}\right]. \tag{3}$$

For a given objective function, the Lagrange multipliers can be chosen to eliminate expensive gradient calculations. For example, when the system has design independent loading conditions, the Lagrange multipliers can be chosen as follows:

$$\lambda_f^T k^{ff} = -\frac{\partial \pi}{\partial T^f} \quad \text{and} \quad \lambda_p^T = 0, \tag{4}$$

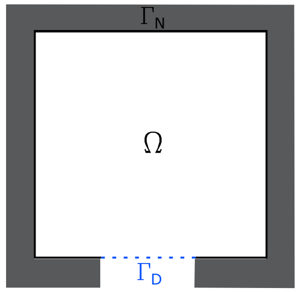
This simplifies Eqn. (3), requiring an inexpensive calculation which can be evaluated after solving a system of linear equations for λ_f:

$$d\Theta_e = \lambda_f^T \frac{\partial k^{ff}}{\partial \rho_e} T^f. \tag{5}$$

The optimization algorithm used in all subsequent studies is the Method of Moving Asymptotes algorithm (MMA).[7] The design domain is descretized into a regular mesh consisting of 10,000 elements (100 x 100). The volume fraction of conductive material is restricted to 20% of the domain volume and the minimum radius of the conductive path (i.e., solid material) is restricted to 5% of the domain width.

II. Homogeneous Heating Domain

The first set of numerical experiments are based on a power system with distributed heating. Consider the homogeneously heated design domain presented in Fig. 1.



$$\nabla \cdot (k\nabla T) + q = 0 \text{ on } \Omega, \tag{6}$$
$$T = 0 \text{ on } \Gamma_D, \tag{7}$$
$$(k\nabla T)\cdot \mathbf{n} = 0 \text{ on } \Gamma_N, \tag{8}$$

Figure 1: Homogeneously heated design domain.

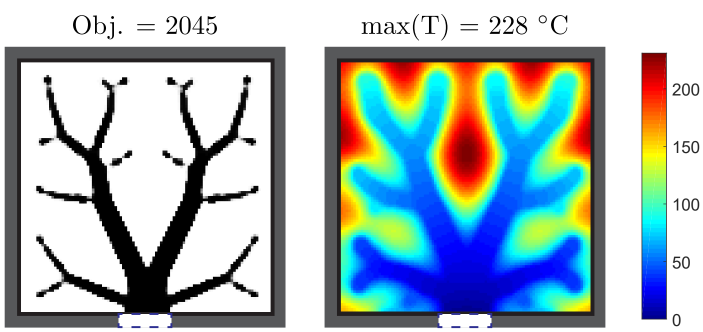
The design domain, Ω, is bounded by the solid black line and is governed by Eqn. (1), where there is a uniform heat flux, q, and the thermal conductivity is defined by k. The fixed temperature Dirichlet boundary, Γ_D, is set to zero, and the heat flux is restricted on the adiabatic Neumann boundary, Γ_N.

A. Thermal Compliance Optimization

First, consider the following thermal compliance optimization problem. This objective function is the product of element-wise heat generation and temperature. This is a popular measure due to its simple and inexpensive sensitivity calculation. The Lagrange multiplier, λ_f, is equivalent to T^f, thereby eliminating the need to solve for the multiplier. Consider the following typical optimization problem:

$$\begin{aligned} \min_{\mathbf{x}}.\quad & \Theta(\mathbf{x}) \leq \int Tq\, dA \\ \text{s. t.}\quad & V(\mathbf{x}) = V_p \\ & R(\mathbf{x}) \geq R_{\min}. \end{aligned} \tag{9}$$

The optimization result is presented in Fig. 2.



a) Topology b) Temperature

Figure 2: Minimum compliance heat spreader.

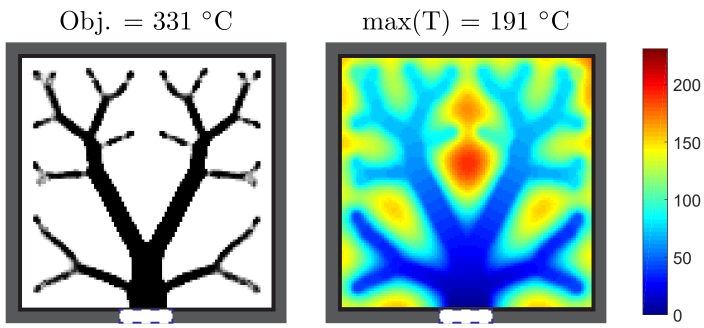
This particular problem formulation for conductive heat transfer has been solved by numerous researchers.[8,9] Though the thermal compliance metric successfully reduces the temperature of the domain through the design of the passive heat spreader, some system design problems may require instead the optimal heat spreader design to reduce the domain temperature as much as possible. To address this, the next optimization study will minimize the maximum temperature on the design domain.

B. Temperature Optimization

For a small increase in computational expense, a temperature objective can be utilized instead of thermal compliance. Specifically, the objective is to minimize the maximum temperature on the domain. Since the the derivative of the maximum function is non-smooth, a p-norm approximation is used. For large p, this measure approximates the infinity norm. Consider the following formulation:

$$
\begin{aligned}
\min_{\mathbf{x}}. \quad & \Theta(\mathbf{x}) = ||T||_p \, dA \\
\text{s. t.} \quad & V(\mathbf{x}) \leq V_p \\
& R(\mathbf{x}) \geq R_{\min}.
\end{aligned} \tag{10}
$$

The general form of the sensitivity equation for this problem can also be reduced to the form of Eqn. (5). An increase in computational expense arises from the need to solve for λ_f. However, this computation is relative inexpensive as it is the solution to a system of linear equations. The optimal topology and temperature map are presented in Fig. 3.



a) Topology b) Temperature

Figure 3: Minimum maximum temperature heat spreader design.

It is clear when using this approximation that the temperature on the domain is reduced significantly when compared to the thermal compliance minimization. Note that the p-norm approximation does not match the maximum temperature, however, the calculated gradients succeed in directing the optimization towards a lower temperature domain. The reduction in temperature justifies the small increase in computational expense required for the calculation of the Lagrange multiplier, λ_f. One motivation for using the compliance metric is to ensure the optimal structure is thermally stiff, or is a structural with low thermal compliance. An alternative optimization formulation may include both compliance and temperature considerations.

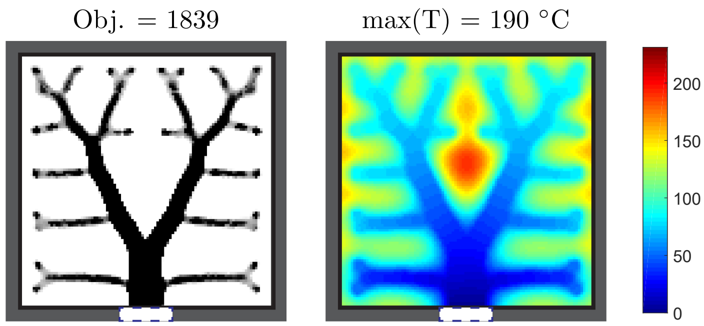
C. Thermal Compliance with Temperature Constraints

To design a thermally stiff heat spreader with a lower temperature profile, consider the following formulation:

$$
\begin{aligned}
\min_{\mathbf{x}}. \quad & \Theta(\mathbf{x}) = \int Tq \, dA \\
\text{s. t.} \quad & V(\mathbf{x}) \leq V_p \\
& R(\mathbf{x}) \geq R_{\min} \\
& T(\mathbf{x}) \leq T_{\max}.
\end{aligned} \tag{11}
$$

Thermal compliance is minimized subject to a temperature constraint. To force the optimization algorithm to obtain an better solution, the maximum temperature is set to the solution value of the temperature minimization optimization, 191°C from Section II B above. The p-norm constraint is modified to be the difference between the nodal and maximum temperature to ensure a more accurate constraint calculation. The sensitivity of this constraint with respect to each design variable is calculated using the adjoint method. It follows a similiar form to that of the objective sensitivity. The optimization results obtained with this formulation are illustrated in Fig. 4.

When enforcing the temperature constraint, the optimization algorithm found a solution that had both improved stiffness and reduced temperatures on the domain. This came at an increased computational cost per iteration compared to the temperature minimization problem, but also resulted in a better solution.



a) Topology b) Temperature

Figure 4: Minimum compliance heat spreader with temperature constraints.

III. Discrete Heating Optimization

Homogeneous heating is a useful representation for certain power electronics systems where the heat generation is dispersed evenly throughout the entire domain. Many power electronics systems, however, cannot be represented accurately using a homogeneous heating model. Discrete lumped heat sources should be accounted for. In this section topology optimization will be investigated for printed circuit boards, where heat sources are now discrete and temperature constraints play an important role in ensuring the proper operation of the electrical devices. Consider the following arbitrary asymmetric circuit layout shown in Fig. 5.

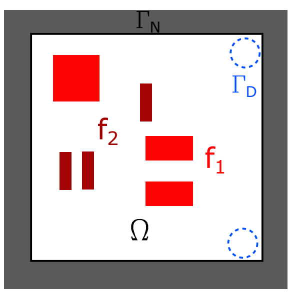


Figure 5: Example printed circuit board design domain.

Table 1: Representative component specifications

Component type	Power Loss	Max Temperature
f_1	1 W	150 °C
f_2	.2 W	100 °C

This domain has two types of heating components, f_1 and f_2, which have unique temperature constraints defined in Table 1. On the domain there are now two heat sinks at which the temperature is held fixed to zero.

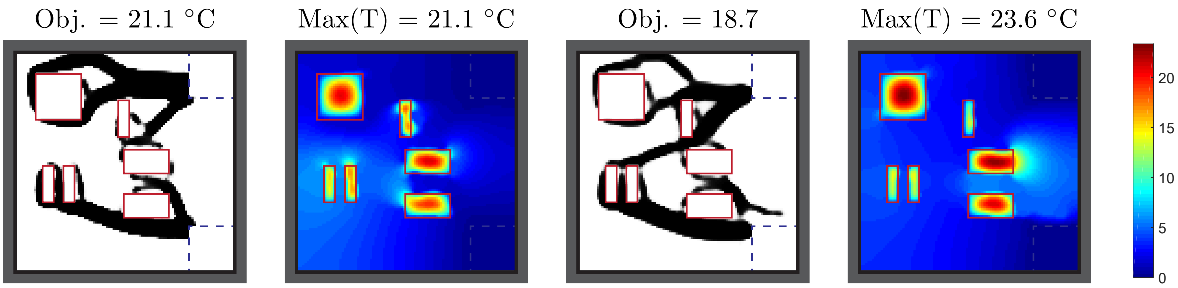
A. Thermal Compliance with Temperature Constraint Study

This first case study will consider the thermal compliance minimization problem with a temperature constraint. To obtain a good maximum value for the temperature constraint, a temperature minimization problem is solved as was done in the homogeneous heating studies, defined in Eqn. (12). The maximum temperature obtained from this optimization is then used as an upper temperature constraint in the thermal compliance problem, defined in Eqn. (13).

$$\begin{aligned} \min_{\mathbf{x}}. \quad & \Theta(\mathbf{x}) = ||T||_p \, dA \\ \text{s. t.} \quad & V(\mathbf{x}) \leq V_p \\ & R(\mathbf{x}) \geq R_{\min}. \end{aligned} \tag{12}$$

$$\begin{aligned} \min_{\mathbf{x}}. \quad & \Theta(\mathbf{x}) = \int Tq \, dA \\ \text{s. t.} \quad & V(\mathbf{x}) \leq V_p \\ & R(\mathbf{x}) \geq R_{\min} \\ & T(\mathbf{x}) \leq T_{\max}. \end{aligned} \tag{13}$$

The thermal compliance minimization problem, defined in Eqn. (13), requires the solution of two adjoint problems: one for the objective function sensitivities, and one for the constraint function sensitives. The maximum temperature from the solution of Eqn. (12), 21.1° C, is used as the $T_{\max}$ value for Eqn. (13). Using this value the appropriate sensitivities are calculated. The optimization solutions for both problems are presented in Fig. 6.



a) Topology b) Temperature c) Topology d) Temperature

Temperature Minimization Compliance Minimization

Figure 6: Optimization solution for minimum thermal compliance.

Using the temperature minimization objective, the temperature of the electrical components is far below that of their maximum operating constraints (100 & 150°C respectively). The maximum temperature obtained here was used as a constraint for the compliance-based optimization. For the thermal compliance minimization problem, the optimizer was unable to meet the temperature constraint, 21.1°C. Yet, the solution obtained has a relatively low temperature profile.

B. Temperature Matching Optimization

Since the resistance properties of a circuit are temperature dependent, there are cases where matching the temperature between two or more components is necessary to ensure proper operation, (e.g., in the operation of photo-coupler). With this consideration, the objective function is to match the average temperature of two of the f_2-type components. The optimization problem is formulated as:

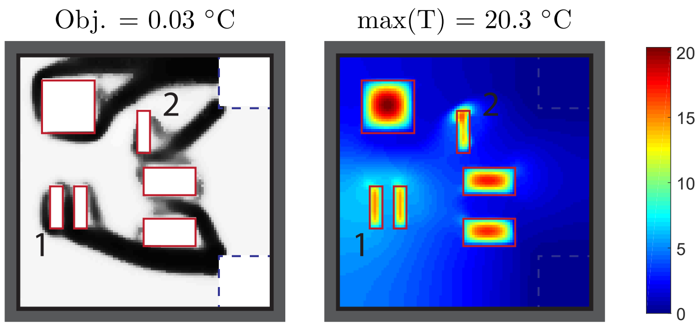
$$
\begin{aligned}
\min_{\mathbf{x}}. \quad & \Theta(\mathbf{x}) = \left(\frac{1}{v_2} \int_2 T \, dA - \frac{1}{v_1} \int_1 T \, dA \right)^2 \\
\text{s. t.} \quad & V(\mathbf{x}) \leq V_p \\
& R(\mathbf{x}) \geq R_{\min} \\
& T(\mathbf{x}) \leq T_{\max},
\end{aligned}
\tag{14}
$$

where the squared difference between the mean temperatures is given by the sum of the component temperatures over the number of component elements, v_1 and v_2. The solution to this formulation is presented in Fig. 7.

As expected, the topology optimization algorithm converged on a solution where the temperature difference between the components approached zero. It achieved this level of performance while satisfying the temperature constraint.

C. Temperature Separation Optimization

Another problem of interest is to design where hot and cold spots are located on the domain. For example, electrical loss and circuit damping are competing objectives that are both a function of electrical resistance and hence temperature. Topology optimization can be used to direct heat flow to areas where temperature may not be a major concern, and can help increase the damping of the local circuit electrical traces (sometimes



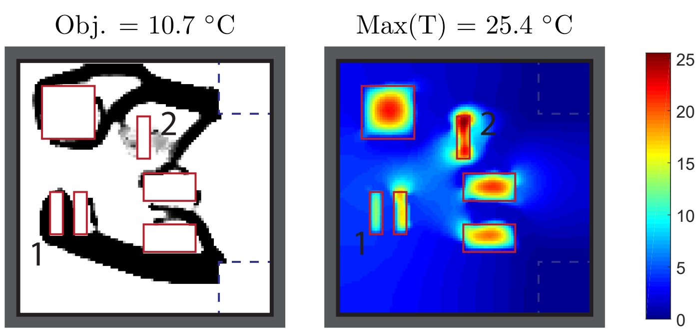
a) SIMP topology b) SIMP temperature

Figure 7: Optimization solution for temperature matching objective.

desirable). One approach for modeling this design problem is given by the following optimization problem:

$$\begin{aligned} \min_{\mathbf{x}}. \quad & \Theta(\mathbf{x}) = \frac{1}{v_2}\int_2 T\,dA - \frac{1}{v_1}\int_1 T\,dA \\ \text{s. t.} \quad & V(\mathbf{x}) \leq V_p \\ & R(\mathbf{x}) \geq R_{\min} \\ & T(\mathbf{x}) \leq T_{\max}, \end{aligned} \tag{15}$$

where the goal is to maximize the temperature difference between two locations on the circuit board. The results of this study are presented in Fig. 8.



a) SIMP topology b) SIMP temperature

Figure 8: Optimized temperature difference solution.

This time the optimizer found a solution where the temperature difference between two points on the domain is maximized. This particular formulation on two components was chosen to illustrate how the solution agrees with intuition, where component 1 is connected to the heat sink by conductive material and component 2 is not. Note, however, that a small amount of conductive material is placed around component 2 to ensure the temperature constraint is met. This type of formulation may be used in a more practical sense to direct heat to spots of the circuit board that are not sensitive to temperature while satisfying temperature constraints on critical components and materials.

Conclusion

Through this investigation, topology optimization techniques were used to optimize various problem formulations pertinent to electronics design. SIMP-based topology optimization was used to solve a variety of problems with distinct constraints and objective functions. Changes in the objective function and constraint choices caused slight changes in heat spreader structure, which lead to large changes in temperature profile.

It was observed that many designs preferred partially-defined material on the domain. Replicating these characteristics in physical prototypes is an ongoing area of research. In this work, several case studies were presented to demonstrate how topology optimization can be used to design more than minimum temperature domains. When analyzing the optimal solutions obtained in these case studies, the results did not always agree. For example, the minimum temperature solution had higher temperatures than the temperature matching optimization solution. This discrepancy may be due to the amount of gray-scale material present on the domain, or the result of the local convergence properties of the MMA algorithm. Investigating global search methods, such as multi-start strategies, may reduce the discrepancy between the results. In addition to algorithm modification, another area of future work is designing the temperature on a domain for high-density circuits. This may lead to interesting results due to stronger analysis and design coupling. Investigating the integration of these formulations in electronics packaging, where the ability to obtain a desired surface temperature may lead to improved component placement for electrical efficiency, is another important topic of future study.

Acknowledgments

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References

[1] Sigmund, O. and Maute, K., "Topology Optimization Approaches," *Structural and Multidisciplinary Optimization*, Vol. 48, No. 6, December 2013 2013, pp. 1031–1055.

[2] Zhuang, C., Xiong, Z., and Ding, H., "A Level Set Method for Topology Optimization of Heat Conduction Problems Under Multiple Loading Cases," *Computer Methods in Applied Mechanics and Engineering*, Vol. 196, January 2007, pp. 1074–1084.

[3] Dede, E. M., "Simulation and Optimization of Heat Flow via Anisotropic Material Conductivity," *Computational Material Science*, Vol. 50, No. 2, November 2010, pp. 510–515.

[4] Dede, E. M., Schmalenberg, P., Nomura, T., and Ishigaki, M., "Design of Anisotropic Thermal Conductivity in Multilayer Printed Circuit Boards," *IEEE Transactions on Components, Packaging and Manufacturing Technology*, Vol. 5, No. 12, December 2015, pp. 1763–1774.

[5] Dede, E. M., Schmalenberg, P., Wang, C., Zhou, F., and Nomura, T., "Collection of Low-Grade Waste Heat for Enchanced Energy Harvesting," *AIP Advances, In-Press*, 2016.

[6] Matsumori, T., Kondoh, T., Kawamoto, A., and Nomura, T., "Topology Optimization for Fluid-Thermal Interaction Problems Under Constant Input Power," *Structural and Multidisciplinary Optimization*, Vol. 47, No. 4, April 2013, pp. 571–581.

[7] Svanberg, K., "The Method of Moving Asymptotes – A New Method for Structural Optimization," *Internation Journal for Numerical Methods in Engineering*, Vol. 24, 1987, pp. 359–373.

[8] Gersborg-Hansen, A., Bendsoe, M. P., and Sigmund, O., "Topology Optimization of Heat Conduction Problems Using the Finite Volume Method," *Structural and Multidisciplinary Optimization*, Vol. 31, March 2006, pp. 251–259.

[9] Burger, F. H., Dirker, J., and Meyer, J. P., "Three-Dimensional Conductive Heat Transfer Topology Optimization in a Cubic Domain for the Volume-To-Surface Problem," *International Journal of Heat and Mass Transfer*, Vol. 67, December 2013, pp. 214–224.