Simultaneous Structural and Control System Design for Horizontal Axis Wind Turbines

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Wind energy is proving to be a promising energy source to complement conventional energy systems in meeting global energy demands, and is currently one of the fastest growing renewable energy sources. Modern wind turbines are large, flexible structures operating in uncertain environments. Power capture and economic value increase with turbine size, leading to steadily increased turbine size over the last three decades. Along with larger size comes intensified structural loads, presenting challenges in mechanical system design. One of these challenges is the dynamic deflection of structural components. These passive system dynamics interact with the active control of wind turbine energy generation. Because of this interaction, addressing the physical and control system design of these devices in a comprehensive manner is vital to ensuring maximum energy extraction, system reliability, and other critical metrics. A large portion of existing work has aimed to increase energy production through optimal control system design (through some combination of rotor speed and pitch control). This strategy treats physical system design as a fixed entity, overlooking potential gains. Further performance increases can be realized through a broader systems approach where physical and control system design are tackled simultaneously. This approach, known as co-design, can capitalize on the synergy that exists between passive system dynamics and active control to increase performance further. In this article a new method for wind turbine design is presented that produces system-optimal results by accounting for the coupling between plant system and control system design. A case study is presented that demonstrates significant performance improvements over conventional sequential design approaches.

Nomenclature

P_m	Rotor power, W
v	Incoming wind speed, m/s
$\mathbf{x_p}$	Plant design vector
F_T	Thrust on the rotor, N
T_r	Rotor torque, N·m
J_r	Rotor interia, kg·m^2
K_r	Rotor torsional stiffness, N·m/rad
ω_r	Rotor speed, RPM
β	Blade pitch angle, deg
T_g	Generator torque measured on low speed side (LSS), N·m
$T_g{}^{hss}$	Generator torque measured on high speed side (HSS), N·m
H_t	Tower top height, m
R	Turbine blade radius, m
R_h	Turbine blade hub radius from rotor axis, m
C_p	Rotor power coefficient
C_q	Rotor torque coefficient

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β	Blade pitch angle, rad
λ	Tip speed ratio
ρ	Air density, Kg/m^3
η_g	Gear reduction ratio

I. Introduction

The operating regimes for wind turbine systems have traditionally been categorized in three operational zones: *Zone 1*: below cut-in wind speed (i.e., speeds below the minimum required to produce useful power), *Zone 2*: between cut-in and cut-out wind speeds (cut-out speed is the speed at which turbine operation must be modified to prevent damage), and *Zone 3*: above the cut-out wind speed. Wind turbines are designed to provide optimum power at the rated wind speed, which is in Zone 2. Efforts aimed at improving dynamic turbine performance in Zone 2 are often focused on increasing power extraction using some combination of rotor torque and speed control, as well as blade pitch control.[1] Addressing control system design without considering the potential synergy of simultaneous physical design modifications does not lead to the best possible system performance. This article presents an investigation that compares conventional sequential design (i.e., performing control system design after physical system design is complete) methods applied to horizontal-axis wind turbines to integrated design methods that consider control and physical system design together (e.g., co-design). Results indicate a significant performance increase between the sub-optimal sequential design result and the system-optimal co-design result.

One widely-used model for wind turbine rotor power, P_m, is given by:[2]

$$P_m = \frac{1}{2}C_p(\lambda, \beta)\rho\pi R^2 v^3 \tag{1}$$

where C_p is the power coefficient, which is a non linear function of blade tip speed ratio (λ) and blade pitch angle (β). ρ is the air density, R is blade length, and v is the wind speed (assumed here to be uniform over the entire swept rotor area). For a given physical turbine design and wind speed in Zone 2, the power capture maximization problem reduces to tracking the optimal power coefficient (C_{p_*}) by controlling the blade-tip speed ratio and blade pitch angle.

Thiringer and Linders[2] presented an early investigation of power capture maximization via rotor speed control (in turn controlling the tip speed ratio λ) in a variable-speed, fixed-pitch machine configuration. They controlled the rotor speed trajectory to track the maximum power coefficient over the range of operating wind speeds in Zone 2. They also implemented the control strategy on a physical wind conversion system and obtained results at two different geographic sites. Experimental results showed significant improvement in power capture compared to a fixed rotor speed approach. Narayana and Putrus[3] extended this idea by using artificial neural networks.[4] They used a Nonlinear Autoregressive Moving Average (NARMA) neural network model to provide the sensorless prediction of wind speed. By using this wind speed prediction, they tracked optimal rotor speed for varying wind speeds. Dang et al.[5,6] used Model Predictive Control (MPC) for maximum power capture of wind power below rated wind speed. Maximum power was obtained by regulating shaft speed to track the optimal trajectories closely. Burnham[7] proposed a novel way of optimal power tracking by manipulations on the generator side. He showed that by adding a variable external resistance to the rotor of an induction generator used in a wind turbine, it is possible to manipulate the torque-speed curve and control the output power. Kusiak et al.[8] presented an intelligent wind turbine control system based on models integrating the following three approaches: data mining, model predictive control, and evolutionary computation. They proposed a multi-objective model involving five different weighted objectives. These weights were adjusted in response to the variable wind conditions and operational requirements.

The above studies focus on rotor speed or torque control. An alternative approach is to use blade pitch control. For example, Namik and Stol[9] proposed a method based on individual blade pitch control that improved power output performance for onshore and offshore wind turbines. They showed that using individualized blade pitch control enhances wind disturbance rejection, helps reduce structural tower loads, and improves power capture.

Tower mass correlates strongly with structural system cost. The need to reduce mass and cost competes with the need to construct taller towers to improve energy capture. Increasing height while targeting lower mass designs results in lighter-weight towers with significant elastic compliance. This increased compliance intensifies the risk of aeroelastic instabilities, adding to design and reliability challenges, and hindering efforts to improve energy capture.[10] The coupling between structural dynamics and control of the turbine and generator, which is stronger for taller towers, motivates the development design approaches that simultaneously address structural and control system design to account for (an even capitalize on) control-structure interaction. This article presents a design study that accounts for flexibility using a finite-element model, and control-structure interaction using a simultaneous structure-control design approach.

It is clear from the literature that most efforts have been focused on individual wind turbine design disciplines or objectives, such as optimal control for power production,[11] control for load alleviation or structural design,[12, 13, 14] optimization for strength or system weight,[15, 16] and blade design for improved efficiency.[17, 18] There appears to be a strong need to solve integrated aeroservohydroelastic problems in wind energy domain, as identified by Jonkman,[19] to obtain system-optimal designs. A tighter integration between physical system (plant) and control system design must be established at a much earlier phase in design process, accounting for the coupling between plant and control system design. With this underlying motivation, we propose a simultaneous approach for plant and control system co-design to generate system optimal solutions to power extraction problem that outperform single-discipline design results. A co-design optimization formulation is proposed, and results are compared to those generated using convention sequential design. The co-design formulation well-suited for this problem, in the sense that it opens up design possibilities not accessible when treating plant and control design separately.

II. System Co-Design

Co-design is a class of design optimization methods for actively-controlled dynamic systems. Design of physical systems and their associated control systems are often coupled tasks; design methods that manage this interaction explicitly can produce system-optimal designs, whereas conventional sequential processes (i.e., plant design followed by control design) may not.[20] In this section, we review the sequential design process, followed by a discussion of two co-design formulations: nested and simultaneous.

A. Sequential System Design

In design practice, the sequential design approach is used most often when developing actively controlled engineering systems. This involves designing the physical system first, and then designing the control system without modifying the plant design. When optimization is used, the sequential approach produces optimal solutions with respect to individual disciplines, plant and control design, but normally will not produce a system-optimal solution. The mathematical formulation for sequential system design includes both the plant and control design optimization problem. Here the plant design optimization problem is formulated as follows:

$$\begin{aligned} \min_{\mathbf{x_p}} \quad & \psi(\boldsymbol{\xi}(t), \mathbf{x_c}, \mathbf{x_p}) \\ \text{s.t.} \quad & \mathbf{g_p}(\boldsymbol{\xi}(t), \mathbf{x_p}) \leq \mathbf{0}, \end{aligned} \tag{2}$$

where, $\mathbf{x_p}$ is the vector of plant design variables, $\psi(\cdot)$ is the plant design objective, and $\mathbf{g_p}(\cdot)$ is the vector of plant constraint functions and $\mathbf{x_c}$ is the control design vector. The solution to Prob. (2)—i.e., the optimal plant design vector $\mathbf{x_{p_*}}$—is used as the basis for the optimal control design problem. The objective and constraint functions in the optimal control problem depend on $\mathbf{x_{p_*}}$, but its value is held fixed during the solution of optimal control problem:

$$\begin{aligned} \min_{\mathbf{x_c}} \quad & \psi(\boldsymbol{\xi}(t), \mathbf{x_c}, \mathbf{x_{p_*}}) \\ \text{s.t.} \quad & \mathbf{g_p}(\boldsymbol{\xi}(t), \mathbf{x_{p_*}}) \leq \mathbf{0} \\ & \dot{\boldsymbol{\xi}}(t) - \mathbf{f}(\boldsymbol{\xi}(t), \mathbf{x_c}, \mathbf{x_{p_*}}) = \mathbf{0}. \end{aligned} \tag{3}$$

The last term in Prob. (3) represents system dynamics, where $\boldsymbol{\xi}(t)$ are time dependent system states and $\mathbf{f}(\cdot)$ is the system derivative function.

The sequential design problem may be formulated in several ways. One approach for characterizing the difference between these formulations is to identify the nature of the objective function in each problem. Allison and Herber[21] presented a taxonomy of sequential design formulations. In much of the literature a distinct objective function is formulated for the plant design problem. The plant objective in this case is often an approximation of the dynamic system objective used in the optimal control problem. In the formulation presented here, we assume that the objective function for the plant and control design problems is identical. The only difference is what varies in each problem formulation.

In more complete co-design formulations, the plant design depends on state. For example, stress values in a turbine tower or blades depend on system state. While both the plant objective function and constraints will depend directly on state, only the objective function will depend directly on control design. Plant constraints, however, will depend indirectly on control design since state is influenced by control design. A co-design problem formulated in this way exhibits bi-directional coupling, i.e., plant design depends on control design, and vice versa. One consequence of plant constraints depending on state is the need to include plant constraints in the control optimization problem. Otherwise feasibility issues may arise.

Several options exist for solving the optimal control problem. A classical or 'indirect' approach is to apply optimality conditions such as Pontryagin's Maximum Principle (PMP),[22,23] and then solve for the optimal control trajectory $\mathbf{x}_{\mathbf{c}*}$ that minimizes $\psi(\cdot)$. If a closed-form solution to the optimality conditions cannot be found, the resulting boundary value problem (BVP) often can be solved numerically. This approach is therefore known as an 'optimize-then-discretize' approach, since optimality conditions are applied first to obtain a BVP, which is then discretized and solved.[24] One significant challenge in utilizing indirect optimal control methods in co-design is the need to satisfy inequality plant constraints. This is not possible in the general case. Other methods are needed that are more naturally suited for solving Prob. (3).

Optimal control problems may also be solved using direct methods, where an infinite-dimensional optimal control problem, such as the one given in Prob. (3), is 'transcribed directly' into a finite-dimensional nonlinear program (NLP). The discretized optimization problem can then be solved numerically using appropriate NLP algorithms, and can easily accommodate inequality plant constraints. This approach, known as Direct Transcription (DT),[24] is classified as a 'discretize-then-optimize' method, since discretization is performed before optimization.

B. Nested Co-Design

Allison and Herber[21] identified the nested co-design formulation as a special case of the Multidisciplinary Design Feasible (MDF) formulation. This formulation has two loops: an outer loop solves the plant design optimization problem, and an inner loop generates the optimal control for each plant design considered by the outer loop. The outer loop formulation is:

$$\begin{aligned} \min_{\mathbf{x_p}} \quad & \psi_*(\boldsymbol{\xi}(t), \mathbf{x_c}, \mathbf{x_p}) \\ \text{s.t.} \quad & \mathbf{g_p}(\boldsymbol{\xi}(t), \mathbf{x_p}) \leq \mathbf{0}, \end{aligned} \tag{4}$$

where, $\mathbf{x_p}$ is the plant design vector, $\mathbf{g_p}(\cdot)$ are the plant design constraints, and $\psi_*(\cdot)$ is an optimal value function that depends only on $\mathbf{x_p}$. This optimal value function is evaluated by solving the inner loop optimal control problem, i.e., for a given plant design, it finds the optimal control and returns the objective function value. For every outer loop function evaluation, the inner loop is solved for the optimal control design vector $\mathbf{x}_{\mathbf{c}*}$:

$$\begin{aligned} \min_{\mathbf{x_c}} \quad & \psi(\boldsymbol{\xi}(t), \mathbf{x_c}, \mathbf{x_p}) \\ \text{s.t.} \quad & \mathbf{g_p}(\boldsymbol{\xi}(t), \mathbf{x_p}) \leq \mathbf{0} \\ & \dot{\boldsymbol{\xi}}(t) - \mathbf{f}(\boldsymbol{\xi}(t), \mathbf{x_c}, \mathbf{x_p}) = \mathbf{0}. \end{aligned} \tag{5}$$

As can be seen from above formulation, the plant design is held fixed during the inner loop solution. Plant design constraints $\mathbf{g_p}(\cdot)$ are imposed in both loops to ensure system-level design feasibility. As with sequential system design, the optimal control problem must be solved using an optimization method that can accommodate inequality plant design constraints.

C. Simultaneous Co-Design

The simultaneous co-design problem formulation is:

$$\begin{aligned} \min_{\mathbf{x_p}, \mathbf{x_c}} \quad & \psi(\boldsymbol{\xi}(t), \mathbf{x_c}, \mathbf{x_p}) \\ \text{s.t.} \quad & \mathbf{g_p}(\boldsymbol{\xi}(t), \mathbf{x_p}) \leq \mathbf{0} \\ & \dot{\boldsymbol{\xi}}(t) - \mathbf{f}(\boldsymbol{\xi}(t), \mathbf{x_c}, \mathbf{x_p}) = \mathbf{0} \end{aligned} \tag{6}$$

The solution to Prob. (6) yields the system-optimal design because it accounts for all dynamic system interactions and plant-control design coupling, resulting in a minimum $\psi(\cdot)$ that is lower than what could be achieved using the sequential approach. This formulation is often referred to as the simultaneous co-design method, as plant and control design decisions are made simultaneously.

III. Co-Design for Wind Turbines

In this section we look at some of the underlying physics of wind turbine operation and formulate the co-design problem for wind turbine design. The wind that is incident on the turbine rotor (consisting of blades and shaft) generates a torque on the rotor shaft. This torque drives the generator that produces the output electric power. The rotor torque, as a function of wind speed, v, is modeled as:

$$T_r = \frac{1}{2} C_q(\lambda, \beta) \rho \pi R^3 v^2 \tag{7}$$

where, $C_q(\lambda, \beta)$ is the torque coefficient. Normally $C_q(\cdot) < 1$ due to aerodynamic losses. The torque coefficient and power coefficient $C_p(\lambda, \beta)$ are related by:

$$C_q(\lambda, \beta) = \frac{C_p(\lambda, \beta)}{\lambda} \tag{8}$$

where λ is the tip speed ratio of the blade, defined as the ratio of tip tangential speed (ωR) to wind speed (v). This ratio is of particular significance as it is a key factor that governs the optimal power extraction from wind turbines.

$$\lambda = \frac{\omega R}{v} \tag{9}$$

Our objective here is to maximize power production, defined in Eqn. (1), with respect to plant and control design variables, subject to plant design constraints. Using the above model, this is equivalent to maximizing the power coefficient $C_p(\lambda, \beta)$ across the range of expected wind speeds. The values for power coefficient are typically obtained by performing an analysis using Blade Element Theory (BEM).[25] The power coefficient curves are different for each wind turbine. An example power coefficient curve is illustrated in Fig. 1. This curve was obtained using the following empirical relationship:[2]

$$C_p = 0.5 \left(\frac{116}{\lambda_i} - 0.4\beta - 5 \right) e^{\frac{-21}{\lambda_i}}, \quad \text{where} \quad \lambda_i = \left(\frac{1}{\lambda + 0.08\beta} - \frac{0.035}{\beta^3 + 1} \right)^{-1} \tag{10}$$

is an intermediate value of the tip speed ratio.

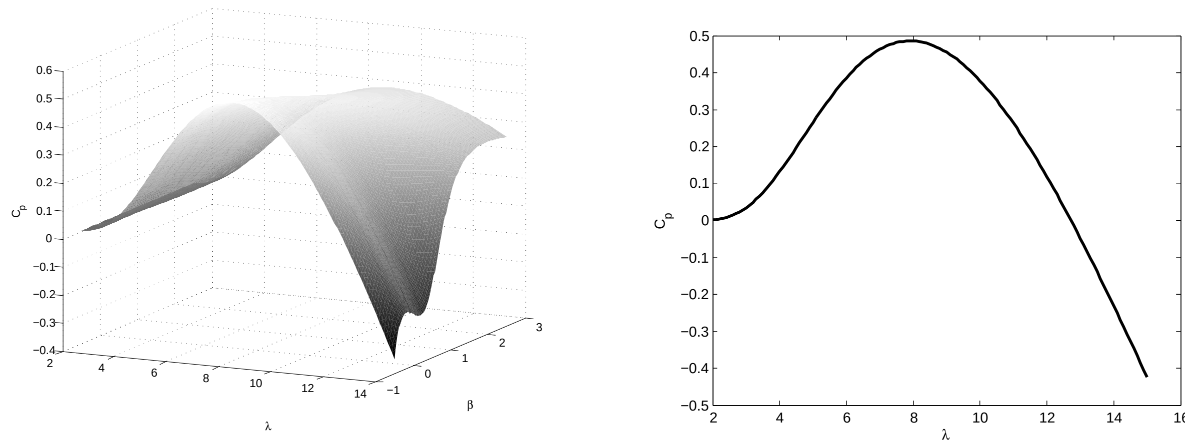
Power production maximization requires that we: 1) Physically design the turbine so that it produces the maximum possible power while satisfying the structural constraints, and 2) Employ an optimal control law that ensures the maximum power coefficient is achieved (and hence maximum power production). The wind turbine considered here is a variable speed, fixed pitch machine. Since β is constant, the turbine should be operated in a way such that the optimum tip-speed ratio (λ) is maintained, which in turn assures maximum power production. This can be achieved by controlling rotor speed via a rotor torque resistance provided by the generator. Consider the following dynamics:

$$J_r \dot{\omega}_r = T_r - K_r \omega_r - T_g \tag{11}$$

where, J_r is the rotor inertia, ω_r is the rotor speed, K_r is the torsional stiffness of the rotor, and T_g is the generator torque as seen on the low speed side (LSS) of the rotor, and acts as a resistance torque on the rotor. This torque T_g is the control input to the system that can be used to regulate the optimal tip speed ratio. The generator torque on the LSS side is related to torque on high speed side (HSS), by the gear ratio:

$$\frac{T_g}{T_g^{hss}} = \eta_g \tag{12}$$

where the rotor and blades are defined as being on the LSS, and generator side is defined as HSS since the generator shaft rotates at higher speed than the rotor.



(a) C_p as function of λ and β

(b) C_p as function of λ for fixed $\beta = 0.2$

Figure 1. Variation of power coefficient (C_p)

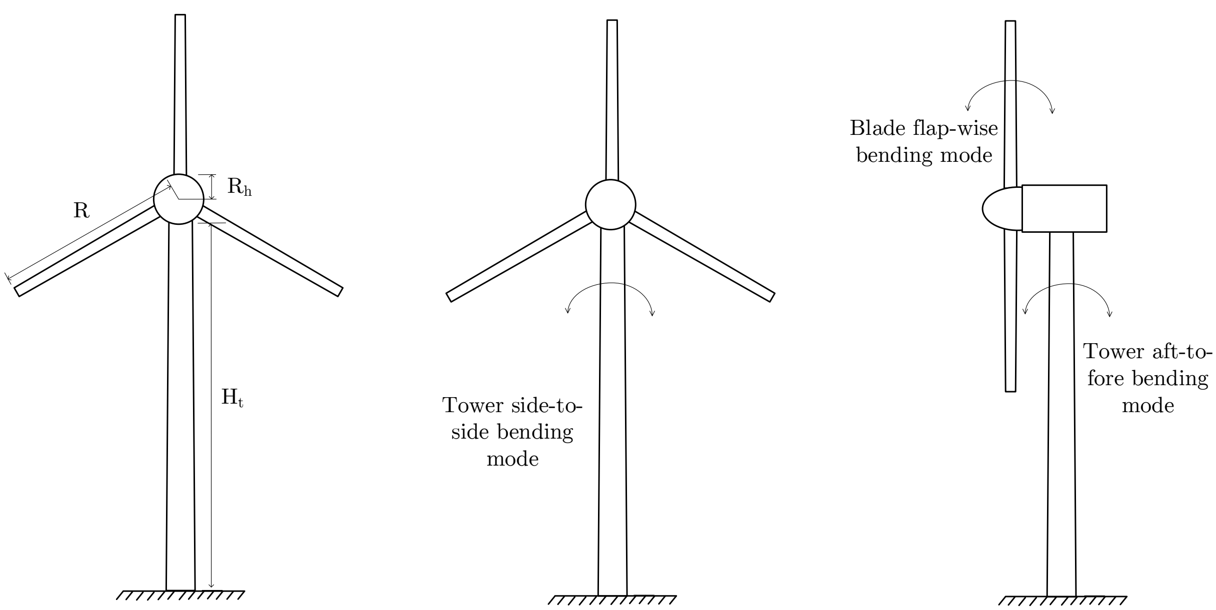


Figure 2. Sketch of wind turbine system with considered modes of vibration

A small subset of the possible plant design variables were chosen for this design formulation based on several factors, including impact on power production and influence on system dynamics (e.g., variables that influence component stiffness, inertia, and damping). The plant design vector used here is:

$$\mathbf{x_p} = \begin{bmatrix} R, & R_h, & H_t \end{bmatrix}^T,$$

where R is the blade length, R_h is the blade hub distance from the rotor axis, and H_t is the tower height from the ground. Please refer to Fig. (2) for a graphical definition of these design variables.

As introduced in Eqn. (11), the control input is: $T_g(t)$, generator torque. In this study open-loop control control is assumed, so the control design variable is the control input trajectory $T_g(t)$. With the analysis model and design variable definitions in place, the design optimization formulations can now be presented. Here we will present three different formulations for comparison: sequential design, nested co-design, and simultaneous co-design.

A. Sequential Design Formulation

In the sequential design formulation we consider the the structural design problem first, followed by the control design. The structural design problem is formulated to maximize the power capture P_m from wind turbine rotor by adjusting the plant design variables ($\mathbf{x_p}$) only. The plant design optimization formulation is:

$$\begin{aligned} \min_{\mathbf{x_p}} \quad & -P_{\max}(\boldsymbol{\xi}(t), \mathbf{x_p}) \\ \text{s.t.} \quad & \|\delta_b(t)\|_\infty - {\delta_b}^{\max} \leq 0 \\ & \|\delta_{t1}(t)\|_\infty - {\delta_{t1}}^{\max} \leq 0 \\ & \|\delta_{t2}(t)\|_\infty - {\delta_{t2}}^{\max} \leq 0 \\ & \dot{\boldsymbol{\xi}}(t) = \mathbf{f}(\boldsymbol{\xi}(t), \mathbf{x_p}, \mathbf{u_e}(t)) \\ & \mathbf{0} < \mathbf{x_l} \leq \mathbf{x_p} \leq \mathbf{x_u}, \end{aligned} \tag{13}$$

where, $P_{\max}(\cdot)$ is the peak power output from the wind turbine for given input wind profile. $\dot{\boldsymbol{\xi}}(t) = \mathbf{f}(\boldsymbol{\xi}(t), \mathbf{x_p}, \mathbf{u_e}(t))$ is the nonlinear structural dynamic model of the wind turbine provided by FAST.[26] FAST is an open-source software developed by National Renewable Energy Laboratory (NREL) for wind turbine design and analysis. FAST includes a finite-element model that approximates structural dynamics. The dynamic model is a nonlinear function of plant design variables and system states. $\mathbf{u_e}(t)$ consists of exogenous inputs, such as thrust force and rotor torque that are input to the system due to incoming wind, which are distinct from control inputs. $\delta_b(t)$ is the flap-wise deflection of the blade tip, $\delta_{t1}(t)$ is the side-to-side deflection of the tower top, and $\delta_{t2}(t)$ is the fore-to-aft tower top deflection. These deflections are constrained by upper bounds ${\delta_b}^{\max}$, ${\delta_{t1}}^{\max}$, and ${\delta_{t2}}^{\max}$, respectively. We employ these deflection constraints as surrogates for preventing fatigue failure. These deflections can be obtained by the running the forward simulation of system dynamic model in FAST. $\mathbf{x_l}$ and $\mathbf{x_u}$ are the lower and upper bounds on the plant design variables.

Once the plant design is optimized, we then proceed onto solving the optimal control problem based on the optimal plant design $\mathbf{x_p}_*$. The optimal control problem is formulated to track the optimal tip speed ratio trajectory $\lambda_*(t)$ for a given input wind speed by regulating the resistance torque ($T_g(t)$):

$$\begin{aligned} \min_{T_g(t)} \quad & \int_0^{t_f} (\lambda(t) - \lambda_*(t))^2 \\ \text{s.t.} \quad & \dot{\omega}_r(t) = -\left(\frac{K_r}{J_r}\right)\omega_r(t) + \frac{1}{J_r}T_r(t) - \frac{1}{J_r}T_g(t). \end{aligned} \tag{14}$$

In sequential system design approach, the plant design problem given by Prob. (13) is solved first, and then optimal control problem given by Prob. (14) is solved while holding the plant design fixed at the value produced by solving Prob. (13). Note that this sequential approach only accounts partially for control-structure interaction. It does not fully capitalize on synergy between plant and control design because the plant design problem is not informed by control design needs. The sequential approach could be iterated to help address this issue, but iterated sequential approaches are typically inefficient and may exhibit convergence problems.[21] Peters et al. introduced the concept of control proxy functions that help inform plant design of control design needs using only a single pass of the sequential approach.[27] In very limited cases the control proxy function approach is capable of producing system-optimal designs. The following two formulations, nested and simultaneous co-design, are able to produce system-optimal designs for a much more general set of system design problems.

B. Nested Co-Design Formulation

As described in Section II, the nested co-design formulation is solved as a two-loop problem. The outer loop solves the problem with respect to plant design variables only. For every function call in the outer loop, an inner-loop problem must be solved. More specifically, the outer loop provides the inner loop with a candidate plant design, and the inner loop problem is to solve the optimal control problem for that particular plant design. The following formulation details the outer-loop formulation for the wind turbine design problem:

$$\begin{aligned} \min_{\mathbf{x_p}} \quad & -P_{\max *}(\mathbf{x_p}) \\ \text{s.t.} \quad & \|\delta_b(t)\|_\infty - {\delta_b}^{\max} \leq 0 \\ & \|\delta_{t1}(t)\|_\infty - {\delta_{t1}}^{\max} \leq 0 \\ & \|\delta_{t2}(t)\|_\infty - {\delta_{t2}}^{\max} \leq 0 \\ & \dot{\boldsymbol{\xi}}(t) = \mathbf{f}(\boldsymbol{\xi}(t), \mathbf{x_p}, \mathbf{u_e}(t)) \\ & \mathbf{0} < \mathbf{x_l} \leq \mathbf{x_p} \leq \mathbf{x_u}, \end{aligned} \tag{15}$$

$P_{\max}(\cdot)$ is an optimal value function; it returns the best possible power capture for a given plant design, which is calculated by solving the optimal control problem. As noted earlier, the plant design constraints are imposed in both the inner and outer loops to system design feasibility. The inner-loop formulation is:

$$\begin{aligned} \min_{T_g(t)} \quad & \int_0^{t_f} (\lambda(t) - \lambda_*(t))^2 \\ \text{s.t.} \quad & \|\delta_b(t)\|_\infty - {\delta_b}^{\max} \leq 0 \\ & \|\delta_{t1}(t)\|_\infty - {\delta_{t1}}^{\max} \leq 0 \\ & \|\delta_{t2}(t)\|_\infty - {\delta_{t2}}^{\max} \leq 0 \\ & \dot{\omega}_r(t) = -\left(\frac{K_r}{J_r}\right)\omega_r(t) + \frac{1}{J_r}T_r(t) - \frac{1}{J_r}T_g(t) \end{aligned} \tag{16}$$

C. Simultaneous Co-Design Formulation

The simultaneous co-design formulation for power capture maximization is:

$$\begin{aligned} \min_{\mathbf{x_p}, T_g(t)} \quad & -P_{\max}(\boldsymbol{\xi}(t), \mathbf{x_p}, T_g(t)) \\ \text{s.t.} \quad & \|\delta_b(t)\|_\infty - {\delta_b}^{\max} \leq 0 \\ & \|\delta_{t1}(t)\|_\infty - {\delta_{t1}}^{\max} \leq 0 \\ & \|\delta_{t2}(t)\|_\infty - {\delta_{t2}}^{\max} \leq 0 \\ & \dot{\boldsymbol{\xi}}(t) = \mathbf{f}(\boldsymbol{\xi}(t), \mathbf{x_p}, \mathbf{u_e}(t)) \\ & \dot{\omega}_r(t) = -\left(\frac{K_r}{J_r}\right)\omega_r(t) + \frac{1}{J_r}T_r(t) - \frac{1}{J_r}T_g(t) \\ & \mathbf{0} < \mathbf{x_l} \leq \mathbf{x_p} \leq \mathbf{x_u}, \end{aligned} \tag{17}$$

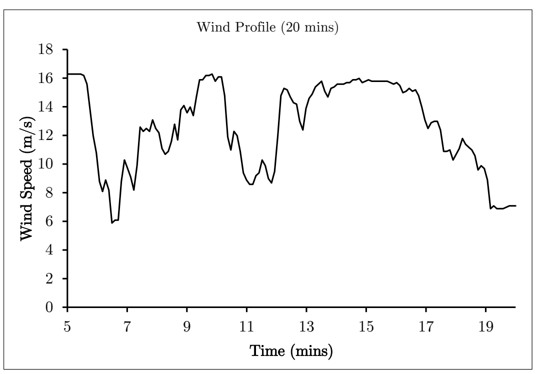
Problem (17) is solved using a discretize-then-optimize approach, such as Direct Transcription.[24] This involves discretizing in time the control and state trajectories, as well as the system dynamics constraints. We seek to solve the Prob. (17) by simultaneously considering plant and control design. The solution provides several important insights based on optimal plant and control design results, improving our understanding of how to improve wind turbine design. Comparison of the nested and simultaneous solution approaches with the sequential approach is presented in the following section.

IV. Results and Discussion

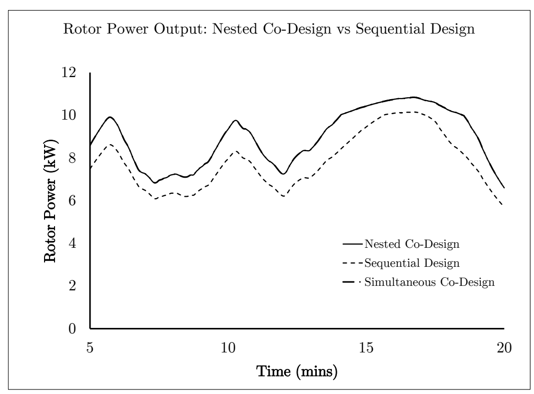
The power output maximization problem was solved for each of the three formulations described above, and the results are reported in this section. Fig. 3(a) shows the input wind profile for which the system was designed. Fig. 3(b) shows the comparison of power output obtained for each of the formulations. The nested and simultaneous approaches both result in the same trajectory, with a peak power of 10.84 kW, which is approximately a 7% increase over the sequential approach (10.14 kW of peak power output). The 7% power increase is very significant when designing higher-capacity wind turbines. This increase in power output can be attributed to the fact that co-design formulations were able to harness the strong interdependence of power output on plant design variables as well as the optimal control torque, to arrive at truly system optimal solution. The plant design variables dictate the size of wind turbine, which has direct impact on power production capability, but bigger turbines are more likely to violate structural constraints. When we solve the co-design problem, the control torque (resistance torque) applied on the rotor, not only helps in maintaining optimal tip speed ratio, but also results in rotor speed reduction. This reduced rotor speed helps in reducing the structural deflections and allows the plant design optimization (outer loop in case of nested formulation) to change plant design so that peak power is optimized. The exploitation of this synergy is only possible if we consider the co-design approach.

Fig. 4(b) illustrates the trajectories of optimal tip-speed ratio for each of the formulations, allowing comparison and extraction of additional insights. It can be observed that in case of sequential design, the plant optimization problem was solved first and optimal control problem was employed to find the optimal tip-speed ratio trajectory for an already optimized plant. The optimal control problem had no flexibility in adjusting plant design to further increase peak power, whereas co-design (both nested and simultaneous) has the ability to: 1) maintain optimal tip-speed ratio and 2) modify the plant design at the same time, helping the optimization algorithm to identify a much better result in the sense of absolute output power.

Fig. 4(a) shows the blade-tip deflection in the flap-wise mode. In all the three formulations blade-tip deflection constraint was observed to be active. Finally, the optimal plant design vector obtained for each of the formulations is listed in Table 1, and bounds imposed on the plant design vector and constraints are listed in Table 2.

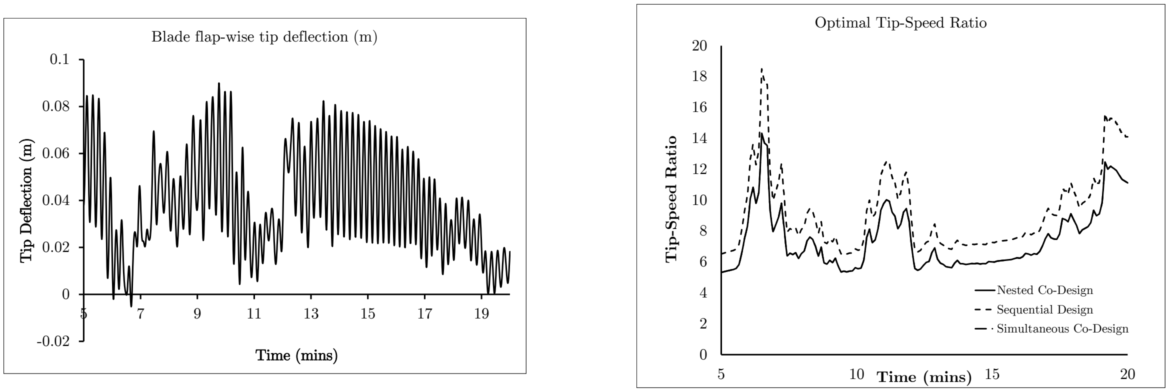


(a) Wind profile



(b) Power output for each of the formulations

Figure 3. Power output comparison for the input wind profile



(a) Blade flap-wise tip deflection

(b) Optimal tip speed ratio

Figure 4. Deflections and Tip speed ratio over time

Table 1. Optimal plant design vector for each of the formulations

Variable	Sequential	Nested	Simultaneous
R, m	3.75	3.00	3.00
R_h, m	0.50	0.50	0.50
H_t, m	30.00	35.95	35.94
Max Power, kW	10.14	10.84	10.83

Table 2. Bounds

Parameter	Value	Unit
${\delta_b}^{\max}$	0.09	m
${\delta_{t1}}^{\max}$	0.01	m
${\delta_{t2}}^{\max}$	0.01	m
$\mathbf{x_l}$	$[2,0.1,30]^\mathrm{T}$	m
$\mathbf{x_u}$	$[8,0.5,50]^\mathrm{T}$	m

V. Conclusion

This article presented a novel approach for optimizing wind turbine design using a simultaneous co-design method to achieve system optimal solutions. Solution of this problem provides significant insight via exploration of design alternatives that are overlooked when using conventional sequential design. The co-design approach presented here (nested & simultaneous) is a balanced formulation in which the deeper treatment is provided to plant model. This type of balanced approach to co-design of engineering systems is a promising step towards enabling more meaningful solutions to multidisciplinary optimization problems for dynamic systems.[21] This approach is in contrast to most of the optimal control formulations in research community which consider a simplistic plant model for control implementation.

References

[1]Pao, L. Y. and Johnson, K. E., "A Tutorial on the Dynamics and Control of Wind Turbines and Wind Farms," *American Control Conference*, IEEE, St. Louis, MO, June 2009.

[2]Thiringer, T. and Linders, J., "Control by Variable Rotor Speed of a Fixed-Pitch Wind Turbine Operating in a Wide Speed Range," *Transactions on Energy Conversion*, Vol. 8, No. 3, 1993.

[3]Narayana, M. and Putrus, G., "Optimal Control of Wind Turbine Using Neural Networks," *UPEC2010*, September 2010.

[4]Rafiq, M. Y., Bugmann, G., and Easterbrook, D. J., "Neural network design for engineering applications," *Computers and Structures*, 2001, pp. 1541–1552.

[5]Dang, D. Q., Wu, S., Yang, W., and Cai, W., "IEEE International Conference on Control and Automation," *IPEC2010*, 2010.

[6]Dang, D. Q., Wu, S., Yang, W., and Cai, W., "Model Predictive Control for Maximum Power Capture of Variable Speed Wind Turbines," *IEEE International Conference on Control and Automation*, Christchurch, New Zealand, December 2009.

[7]Burnham, A., "Variable Rotor-Resistance Control of Wind Turbine Generators," *Power & Energy Society General Meeting*, IEEE, 2009, pp. 1–6.

[8]Kusiak, A., Li, W., and Song, Z., "Dynamic control of wind turbines," *Renewable Energy*, 2010, pp. 456–463.

[9]Namik, H. and Stol, K., "Performance analysis of individual blade pitch control of offshore wind turbines on two floating platforms," *Mechatronics*, , No. 21, 2011, pp. 691–703.

[10]Holley, W. E., "Wind Turbine Dynamics and Control: Issues and Challenges," *American Control Conference*, Denver, CO, June 2003.

[11]Munteanu, I., Cutululis, N., Bratcu, A. I., and Ceanga, E., "Optimization of variable speed wind power systems based on a LQG approach," *Control Engineering Practice*, 2005, pp. 903–912.

[12]Xudong, W., Shen, W. Z., Zhu, W. J., and Sorensen, J. N., "Shape Optimization of Wind Turbine Blades," *Wind Energy*, 2009, pp. 781–803.

[13]Maalawi, K. Y. and Negm, H. M., "Optimal frequency design of wind turbine blades," *Journal of Wind Engineering and Industrial Aerodynamics*, 2002, pp. 961–986.

[14]Soltani, M., Wsniewski, R., Brath, P., and Boyd, S., "Load Reduction of Wind Turbines Using Receding Horizon Control," *International Conference on Control Applications (CCA)*, IEEE, Denver, CO, September 2011.

[15]Negm, H. M. and Maalawi, K. Y., "Structural design optimization of wind turbine towers," *Computers and Structures*, 2000, pp. 649–666.

[16]Nicholson, J. C., *Design of wind turbine tower and foundation systems: optimization approach*, Master's thesis, 2011.

[17]Giguere, P., Selig, M. S., and Tangler, J. L., "Blade Design Trade-Offs Using Low-Lift Airfoils for Stall-Regulated HAWTs," *Journal of Solar Energy Engineering*, 1999, pp. 217–223.

[18]Jureczko, M., Pawlak, M., and Mezyk, A., "Optimisation of wind turbine blades," *Journal of Materials Processing Technology*, 2005, pp. 463–471.

[19]Jonkman, J., "Dynamics of Offshore Floating Wind Turbines - Model Development and Verification," *Wind Energy*, 2009, pp. 459–492.

[20]Reyer, J., Fathy, H., Papalambros, P., and Ulsoy, A., "Comparison of combined embodiment design and control optimization strategies using optimality conditions," *International Design Engineering Technical Conferences, JE Renaud ed.*, ASME, 2001, pp. 1023–1032.

[21]Allison, J. and Herber, D., "Multidisciplinary Design Optimization of Dynamic Engineering Systems," *AIAA Journal*, 2013, To appear.

[22]Pontryagin, L. S., "The Mathematical Theory of Optimal Processes," *Interscience*, 1962.

[23]Bryson, A. E. and Ho, Y. C., *Applied Optimal Control: Optimization, Estimation and Control*, Taylor & Francis, 1975.

[24]Biegler, L. T., *Nonlinear Programming: Concepts, Algorithms, and Applications to Chemical Processes*, SIAM, 2010.

[25]Wood, D., "Blade Element Theory for Wind Turbines," *Small Wind Turbines*, Green Energy and Technology, Springer London, 2011, pp. 41–55.

[26]Jonkman, J. M. and Buhl, M. L., "FAST User's Guide," *Technical Report - NREL/EL-500-38230*, August 2005.

[27]Peters, D., Papalambros, P., and Ulsoy, A., "Control Proxy Functions for Sequential Design and Control Optimization," *Journal of Mechanical Design*, , No. 9, 2011, pp. 091007.