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PLANT-LIMITED CO-DESIGN OF AN ENERGY-EFFICIENT COUNTERBALANCED ROBOTIC MANIPULATOR

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ABSTRACT

Modifying the design of an existing system to meet the needs of a new task is a common activity in mechatronic system development. Often engineers seek to meet requirements for the new task via control design changes alone, but in many cases new requirements are impossible to meet using control design only; physical system design modifications must be considered. Plant-Limited Co-Design (PLCD) is a design methodology for meeting new requirements at minimum cost through limited physical system (plant) design changes in concert with control system redesign. The most influential plant changes are identified to narrow the set of candidate plant changes. PLCD provides quantitative evidence to support strategic plant design modification decisions, including tradeoff analyses of redesign cost and requirement violation. In this article the design of a counterbalanced robotic manipulator is used to illustrate successful PLCD application. A baseline system design is obtained that exploits synergy between manipulator passive dynamics and control to minimize energy consumption for a specific pick-and-place task. The baseline design cannot meet requirements for a second pick-and-place task through control design changes alone. A limited set of plant design changes is identified using sensitivity analysis, and the PLCD result meets the new requirements at a cost significantly less than complete system redesign.

1 Introduction

Mechatronic systems exhibit tight coupling between physical system (plant) design and control system design, requiring an integrated approach for developing high-performance systems [1]. In conventional sequential mechatronic design the physical system is designed completely before control system design [1–3]; this approach fails to produce system-optimal results [4]. Physical system design should be informed by control system considerations. More integrated design methods are required as the complexity and scale of mechatronic design problems increase.

One approach that accounts fully for plant and control system interactions is co-design, an optimization-based design methodology where physical system (plant) and control system design are considered simultaneously [2, 4–7]. Using this approach we can capitalize on the synergy between physical and control system design decisions to produce designs with superior performance. Fully concurrent co-design, however, can be a challenge to implement. It is computationally intensive, often new system models must be developed, and it is not yet congruent with typical organizational structure. An intermediate approach that produces better results than conventional sequential design with lower computational expense than co-design employs control proxy functions to augment the plant design problem [8]. While the result of this modified sequential approach is not system optimal, the plant design problem is more closely aligned with the overall system design problem.

Repurposing existing mechatronic systems to meet the needs of new applications presents another opportunity to incorporate integrated design approaches into engineering practice. Engineers are often asked to modify existing mechatronic systems to meet the requirements of a new task instead of developing new systems from scratch. This reduces redevelopment and manufacturing costs, capitalizing on past development efforts and organizational expertise. System modifications typically are limited to inexpensive control design changes, avoiding costly physical system changes. At times, the new system application requirements cannot be met through control changes alone; either requirements must be relaxed or physical system modifications must be considered. In the latter case changes must be identified that satisfy requirements at minimum cost. A formal methodology for identifying candidate plant modifications and using co-design to minimize plant modification cost, known as plant-limited co-design (PLCD), was introduced recently [9]. In [9] a robotic manipulator design problem was considered where simplified plant design consisted only of link lengths and cross-section radii. Here a more sophisticated physical system is used. Counterbalance masses help reduce joint torques and provide more design flexibility and potential for lower energy consumption. This expanded plant design problem aids more complete exposition of the PLCD process, including new elements of sensitivity analysis.

1.1 Energy Efficient Robotic Manipulator Design

Robotic manipulators are ubiquitous in modern manufacturing; improved performance and energy efficiency can have important economic and environmental impact [10, 11]. While early manipulator design efforts were focused primarily on reducing task time [12], recent work has concentrated more on energy efficiency improvement. Energy consumption is especially important for mobile robotics [13–18], and techniques learned from these studies, such as trajectory design [15–17] and use of passive dynamics [13, 19], often can be extended to manipulator design [12, 20–22].

Manipulator counterbalance mechanisms [23–25], such as springs or counter masses, can help reduce joint torques and energy consumption. Gravity-balanced manipulators can be designed such that the net torque at each joint in any position is zero under static conditions [26]. Chung and Cho developed a mechanism and control policy for automatically adjusting counterweight–joint distances to accommodate a range of payloads [27], and Lim et al. investigated potential payload capacity increase using counterbalance mechanisms [28].

Cancellation of gravity forces produces near minimal energy consumption for slow movements. When motions are fast, however, dynamic effects increase joint torque requirements due to large counterweight masses [28]. Gravity force cancellation is easy to design for, and this has been used often as a proxy design objective for energy minimization [27–30]. While using this proxy objective simplifies physical system design, resulting systems are suboptimal and do not use minimal energy. This suboptimality is especially pronounced for high-acceleration applications.

Developing robotic manipulators that consume minimal energy while performing practical (i.e., not pseudo-static) industrial tasks demands a more integrated design approach that accounts for physical dynamics. One strategy is to exploit the existing passive dynamics of a physical system to help perform the desired task. This strategy, which is often exhibited in biological systems, has been an inspiration for energy-efficient biomimetic systems. For example, passive dynamic walkers can walk down an incline powered only by gravitational potential energy and without any active joint control [31]. Adding small power sources to replace the force of gravity can enable similar walkers to move on level ground or up inclines with energy efficiency that is magnitudes better than conventional humanoid robots and comparable to the efficiency of human walking [32]. Carefully designed active control can work in concert with passive dynamics when the passive system alone cannot perform the desired task [13, 33, 34]. This approach can work well for optimizing energy efficiency or other performance metrics when passive dynamics are aligned with the desired task. If there is misalignment (e.g, slower time constants), control design alone is insufficient and plant redesign should be considered.

Often when passive dynamics are addressed in the literature the focus is on control system design. Co-design offers a more comprehensive approach for developing a system design tailored to a desired task. Co-design is a more general approach for utilizing passive dynamics that capitalizes on synergy between physical and control system design; it applies to a broader range of systems and requirements and can produce superior system performance. In this article co-design will be applied to the energy minimization of a robotic manipulator performing specified tasks, both for the initial system design and the repurposed design in the PLCD example.

1.2 Plant-Limited Co-Design

As introduced in [9], PLCD is a systematic process for repurposing mechatronic systems to meet requirements of a new application at minimal cost, consisting of four steps:

1. Identify candidate plant modifications
2. Develop a suitable system model
3. Formulate and solve the PLCD optimization problem
4. Verify result and repeat if needed

In the first step we analyze the system to determine which aspects of the plant we should modify. One approach for making this selection is to compute sensitivities that estimate which plant modifications will have the largest impact on system performance. Normally at this stage only a model suitable for control design is available, i.e., it cannot accommodate plant design changes. The second step expands the system model so that it can predict the effect of limited plant changes. When the model is complete we can formulate and solve the PLCD optimization problem. After we arrive at a solution we can verify whether further improvement can be achieved by altering the set of candidate plant modifications. If this is the case, the process can be repeated. These steps are described in more detail below.

1.2.1 Selecting Candidate Plant Modifications

While plant modifications in this article are limited to continuous geometric changes of existing components, they could include topological changes such as the addition, removal, or reconfiguration of components. We are interested in which candidate plant modifications have the greatest influence on requirements for the new system task that were violated using the original plant design. If the complete set of requirements for the new task is $\mathbf{g}_r(\mathbf{p}) \leq \mathbf{0}$, the subset of requirements the original system design is unable to meet is denoted by $\bar{\mathbf{g}}_r(\mathbf{p}) \leq \mathbf{0}$. $\mathbf{p}$ are system model parameters that depend on plant design variables. We compute the sensitivity of violated constraints with respect to plant design variables if a more complete plant model is available, and with respect to $\mathbf{p}$ if not. The rationale here is to identify elements of plant design with the ability to influence system performance as efficiently as possible. Here we use the following sensitivity calculation to estimate this influence:

$$\frac{\partial \bar{g}_{ri}(\mathbf{p})}{\partial p_j}, \quad i = 1,2,\ldots,n_r, \quad j = 1,2,\ldots,n_p \tag{1}$$

where n_r is the number of violated constraints and n_p is the number of model parameters. These terms form the model parameter Jacobian $\mathbf{J}_p$. Large $\partial \bar{g}_{ri}(\mathbf{p})/\partial p_j$ indicates that requirement i is influenced significantly by small changes in p_j. After a set of influential model parameters $\bar{\mathbf{p}}$ is identified based on $\mathbf{J}_p$, a corresponding set of candidate plant changes $\mathbf{x_p}$ that influence $\bar{\mathbf{p}}$ must be established. The system model can then be expanded based on $\mathbf{x_p}$, as described in the next step.

If a cost model is available, a more sophisticated analysis for selecting $\mathbf{x_p}$ can be performed that involves the sensitivity of violated requirements with respect to cost:

$$\left(\frac{\partial \bar{g}_{ri}(\mathbf{p})}{\partial p_j}\right)\left(\frac{\partial C(\mathbf{p})}{\partial p_j}\right)^{-1} = \frac{\partial \bar{g}_{ri}(\mathbf{p})}{\partial c_j}, \quad \begin{array}{l} i = 1,2,\ldots,n_r \\ j = 1,2,\ldots,n_p \end{array} \tag{2}$$

where $C(\mathbf{p})$ approximates the cost of changing model parameters, and c_j represents the cost of changing model parameter p_j. The $\partial \bar{g}_{ri}(\mathbf{p})/\partial c_j$ terms form the cost Jacobian $\mathbf{J}_c$, which can be used to assess which plant modifications could produce the desired performance improvements most economically.

Sensitivity analysis is inherently local, and in some cases may not lead to the best set of candidate plant modifications. Other more sophisticated methods, such as change propagation analysis [35, 36], should be explored as alternative strategies for identifying candidate plant modifications. That said, sensitivity analysis was shown through enumeration to identify the optimal set of candidate plant modifications for the case study presented in [9].

While this step is presented as a component of the larger PLCD process, it may be used alone in conjunction with conventional design methods. For example, once $\mathbf{x_p}$ is identified using sensitivity analysis, engineers can proceed using conventional design methods to modify the plant and then the control system in a sequential manner. This may be sufficient, and the relatively small modeling and analysis investment is appealing, but will not result in a system-optimal design. If this simplified approach is not successful the full PLCD process should be implemented. In addition, if one iteration of PLCD fails to identify a new design that satisfies requirements based on a particular $\mathbf{x_p}$, the criteria for selecting $\bar{\mathbf{p}}$ should be relaxed to increase the dimension of $\mathbf{x_p}$.

1.2.2 System Model Development

Solving the PLCD optimization problem described in step 3 requires a system model that has as independent variables the candidate plant modifications $\mathbf{x_p}$ identified in step 1. Models that predict accurately the results of physical system design changes are challenging to develop, often requiring significant resource investment. Developing a system model that accommodates all possible plant changes is unnecessary (and often impractical) for PLCD problems; reducing the number of candidate plant changes in step 1 is important for curbing model development expense and easing optimization solution difficulty.

One possible modeling approach is to augment the existing control design oriented model used in step 1 with specialized modeling tools that may be used to predict model parameter values. More precisely, we seek a model of the form $\mathbf{p} = \mathbf{a}(\mathbf{x_p})$, where $\mathbf{a}(\mathbf{x_p})$ is an analysis function that computes model parameters as a function of independent plant design variables using computer aided engineering tools such as finite element analysis. This system model structure was described by Allison and Nazari [37].

Reducing the dimension of $\mathbf{x_p}$ via sensitivity analysis or other means may be viewed as a type of model order reduction [38], but instead of reducing state space dimension we seek to reduce design space dimension. In addition, in model order reduction the objective is to reduce the complexity of an existing model, whereas here the objective is restrict model dimension a priori. Techniques for a posteriori design model reduction exist, such as monotonicity analysis [39], but strategies for the a priori design model reduction required in PLCD is an opportunity for future work.

1.2.3 PLCD Problem Formulation and Solution

With a reduced set of candidate plant modifications $\mathbf{x_p}$ and a corresponding plant model now available, the PLCD problem can be formulated as a design optimization problem and solved. Consider the following PLCD optimization problem:

$$\begin{aligned} \min_{\mathbf{x}=\left[\mathbf{x_p}^{\mathrm{T}},\, \mathbf{x_c}^{\mathrm{T}}\right]^{\mathrm{T}}} \quad & \phi(\mathbf{x}) \\ \text{s.t.} \quad & \mathbf{g_p}(\mathbf{x}) \leq 0 \\ & \mathbf{g_r}(\mathbf{x}) \leq 0 \end{aligned} \tag{3}$$

where $\phi(\mathbf{x})$ is a measure of deviation from the original plant design, $\mathbf{g_p}(\mathbf{x})$ are plant design constraints, and $\mathbf{g_r}(\mathbf{x})$ are system performance requirements formulated as inequality constraints. Both sets of constraints are assumed to be given. Note that $\mathbf{g_r}(\mathbf{x})$ represents the full set of system requirements, not just those used in step 1 for identifying candidate plant modifications. In addition, because a new objective function $\phi(\mathbf{x})$ is introduced, the original design objective may be recast as a constraint to ensure adequate performance. The optimization variable $\mathbf{x_p}$ parametrizes the candidate plant modifications, and $\mathbf{x_c}$ are the control design variables. The solution to this problem is a system design that meets performance requirements while requiring minimal modifications to the physical system design.

Several options exist for construction the metric $\phi(\mathbf{x})$. It could be a simple weighted norm that penalizes moving away from the original design $\mathbf{x_0}$ (e.g., $\phi(\mathbf{x}) = \|\mathbf{w} \circ (\mathbf{x_0} - \mathbf{x_p})\|$), or more ideally a cost model that predicts the cost of changing the plant design. In the case study here the cost of plant changes is approximated using component-wise change in mass:

$$\phi(\mathbf{x}) = \sum_{i=1}^{n_c} |m_i(\mathbf{x}) - \hat{m}_i|, \tag{4}$$

where n_c is the number of plant components being modified, $m_i(\mathbf{x})$ is the mass of component i with modification defined by $\mathbf{x}$, and $\hat{m}_i$ is the mass of component i in the baseline design. This approximates the cost of limited plant redesign to the manufacturer. A more complete study would include full lifecycle cost, including use (such as energy and maintenance costs) and end-of-life costs, a subject of current work. Multi-objective optimization may also be performed to explore the tradeoff between cost and system performance for the new application [9]. This tradeoff will be investigated for the counterbalanced robotic manipulator problem presented here.

2 Energy Minimization Using Co-Design

A two-link planar manipulator was selected to demonstrate the PLCD solution process. This manipulator is similar to the one used in [9], but incorporates a counterbalance mechanism that helps to reduce energy consumption further. This topological change also provides greater opportunity for plant design modifications, useful for our study of PLCD. The manipulator used here, shown in Fig. 1a, has two mechanical degrees of freedom (θ_1 and θ_2), and is designed to move a payload at the end effector for a pick-and-place task. Links 1 and 2 are counterbalanced by masses m_{c1} and m_{c2}, respectively. Here we do not assume these masses or their offset distances (ℓ_{c1} and ℓ_{c2}, respectively) are chosen such that gravity forces cancel out. Designs without perfect gravity balance may be more desirable due to dynamic effects.

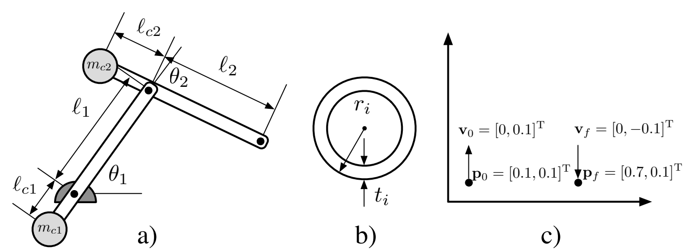


Figure 1: a) Counterbalanced two-link planar manipulator. b) Section view of link i. c) Task A initial and final conditions.

Each link has a constant cross section depicted by Fig. 1b with radius r_i and wall thickness t_i. The complete plant design vector is:

$$\mathbf{x_p} = [\ell_1, \ell_2, \ell_{c1}, \ell_{c2}, m_{c1}, m_{c2}, r_1, r_2]^{\mathrm{T}}.$$

An initial system design is required to demonstrate PLCD, and will be obtained by applying co-design to determine a system-optimal $\mathbf{x_p}$ and $\mathbf{x_c}$ that minimize the energy consumed to perform a specific task while satisfying torque and deflection constraints:

$$\begin{aligned} \min_{\mathbf{x}=[\mathbf{x_p},\mathbf{x_c}]} \quad & E(\mathbf{x}) \\ \text{s.t.} \quad & |\tau_i(\mathbf{x})| \leq \tau_{\text{allow}}, \quad i = 1, 2 \\ & \delta_i(\mathbf{x_p}) \leq \delta_{\text{allow}}, \quad i = 1, 2 \end{aligned} \tag{5}$$

The initial task (Task A) requires moving a 20 kg payload from the initial position $\mathbf{p}_0$ with initial velocity $\mathbf{v}_0$, and $t_f = 2.0$ seconds later place the payload at $\mathbf{p}_f$ with final velocity $\mathbf{v}_f$ (Fig. 1c).

The desired (quintic) trajectory for the joint angles $\mathbf{q_d}(t) = [\theta_1(t), \theta_2(t)]^{\mathrm{T}}$ is calculated based on the task end conditions, as well as the position $\mathbf{p}_I$ and velocity $\mathbf{v}_I$ of an intermediate point along the path between $\mathbf{p}_0$ and $\mathbf{p}_f$. The intermediate point parameterizes the trajectory, and is a design decision to be made in solving Prob. (5). Feedback linearization with a PD controller was used to track $\mathbf{q_d}(t)$ [11], but it was found that inverse dynamics (i.e., calculating joint torque trajectories as a function of $\mathbf{q_d}(t)$ using equations of motion) resulted in very little torque or energy consumption error compared to forward simulation with feedback linearization. Using inverse dynamics reduced simulation expense as well as control design vector dimension:

$$\mathbf{x_c} = [p_{I1}, p_{I2}, v_{I1}, v_{I2}]^{\mathrm{T}}.$$

Here we assume no energy recovery under forward or reverse braking conditions [12–14]. Maximum deflection was calculated using standard beam formulas with a nominal joint torque applied [9]. The deflection constraint can be shown to be active [39], allowing solution for required radius values and subsequent plant design vector reduction via substitution:

$$\mathbf{x_p} = [\ell_1, \ell_2, \ell_{c1}, \ell_{c2}, m_{c1}, m_{c2}]^{\mathrm{T}}.$$

The following nonlinear differential equation [11] was used to model manipulator dynamics:

$$\mathbf{M}(\mathbf{q}, \mathbf{x_p})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{x_p})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}, \mathbf{x_p}) = \boldsymbol{\tau}, \tag{6}$$

where $\mathbf{M}(\mathbf{q}, \mathbf{x_p})$ is the inertia matrix, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{x_p})$ represents centrifugal and Coriolis terms, and $\mathbf{g}(\mathbf{q}, \mathbf{x_p})$ is the gravity vector. The joint torque vector is $\boldsymbol{\tau} = [\tau_1, \tau_2]^{\mathrm{T}}$. In this model joint motor mass, motor rotational inertia, and electrical losses have been neglected. Energy consumption is calculated by integrating mechanical power at each joint, neglecting braking power. Complete details of this model are provided via published MATLAB® code [40].

Co-design is effective at exploiting passive dynamics in mechatronic design. Most past efforts to improve energy efficiency, even those that incorporate passive dynamics, focus on control design efforts. In co-design plant behavior can be tailored to the needs of a specific task simultaneously with control design. While mechatronic design is often approached in an integrated manner, application of co-design to robotic systems has been limited. Ravichandran et al. applied co-design to energy minimization of a counterbalanced manipulator, but used gravity balance as a proxy objective, resulting in a modified sequential process [30]. Park and Asada applied co-design to a manipulator, but used simplified system performance metrics, such as settling time [41]. Here we apply for the first time a full co-design technique to robotic manipulator design to minimize energy consumption computed using a nonlinear dynamic simulation. The result is used as a baseline design for PLCD.

For comparison, an optimal trajectory was computed for a nominal plant design ($\mathbf{x_p} = [1.0, 1.0, 0.3, 0.3, 10, 10]^{\mathrm{T}}$). The energy consumed to perform Task A defined in Fig. 1c for this nominal design was 27.6 Joules, and the corresponding trajectories are illustrated in Fig. 2. The payload is lifted up from $\mathbf{p}_0$ and set down at $\mathbf{p}_f$ (Fig. 2a). Here the maximum torque bound limit was 210 Nm, which was reached by τ_1 (Fig. 2b). The starting point of the torque-speed trajectories is indicated with a circle.

The baseline design for Task A was generated by solving the co-design problem given in Eqn. (5) (with reduced $\tau_{\mathrm{allow}} = 160$ Nm), reducing energy consumption to essentially *zero* for Task A ($E = 5.86 \times 10^{-5}$ Joules). The payload follows a similar path (Fig. 3) to the nominal design, but the torque-speed trajectories are significantly different. Torque curves for both joints remain in either the forward or reverse braking quadrants (i.e., $\tau \times \omega \leq 0$). This result is similar to the design strategy used by Arai and Taichi, who combined actively controlled joints and joints with only braking capability to design a manipulator that could perform tasks similar to those used in this article [42]. Recall that here we assume no braking energy can be recovered (i.e., braking energy dissipated as heat), so negative power under braking does not contribute to the energy consumption metric. In the system-optimal solution presented here only braking occurs at the joints, so no energy is consumed by the joint actuators to perform Task A. The optimal system design is:

$$\mathbf{x_p} = [0.838, 0.711, 0.216, 0.885, 3.89, 20.7]^{\mathrm{T}}$$

$$\mathbf{x_c} = [0.359, 0.163, 0.00829, -0.0249]^{\mathrm{T}}$$

The link lengths are significantly shorter than those presented in [9] for the same task. The counterbalance mechanism enables shorter arms to attain correct passive dynamics, which allows Task A completion with only joint braking. In [9], however, there was greater utilization of passive dynamics as evidenced by torque trajectories that were near zero during much of the simulation. The manipulator in [9] was not counterbalanced and required more energy to perform Task A.

3 Plant-Limited Co-Design of a Robotic Manipulator

The optimal result of the previous section serves as a baseline design for demonstrating PLCD. Here we introduce a new task (Task B) that the manipulator is to perform, ideally with very limited changes. The new task is specified by the following end conditions:

$$\mathbf{p}_0 = [0.5, 1.2]^{\mathrm{T}}\,\mathrm{m}, \quad \mathbf{v}_0 = [-0.1, 0]^{\mathrm{T}}\,\mathrm{m/s},$$

$$\mathbf{p}_f = [0.4, 2.0]^{\mathrm{T}}\,\mathrm{m}, \quad \mathbf{v}_f = [-0.1, 0]^{\mathrm{T}}\,\mathrm{m/s}$$

In Task A the payload vertical position did not change between end conditions, but here it changes by 0.8 m, resulting in a gravitational potential energy increase (for just the payload) of:

$$(20\,\mathrm{kg}) \times (9.81\,\mathrm{m/s^2}) \times (0.8\,\mathrm{m}) = 157\ \mathrm{J}$$

Observing the link lengths from the Task A design, the maximum extension of the manipulator is $0.838 + 0.711 = 1.549$ m, and is unable to perform Task B due to kinematic restrictions ($\mathbf{p}_f$ is unreachable: $\|\mathbf{p}_f\| = 2.040\,\mathrm{m} > 1.549$ m). In the PLCD example given in [9] the Task A design was kinematically (but not dynamically) capable of performing Task B. This new complication of kinematic infeasibility allows explication of new candidate plant modification techniques.

3.1 Task B Co-Design

Before proceeding with candidate plant design change selection and PLCD optimization, here we present the system design that results when solving Eqn. (5) with a complete set of plant design variables for Task B. In other words, this is what results when using co-design to redesign completely the manipulator for Task B. This represents that best possible design for Task B if cost was not a concern, and will be used for comparison to the

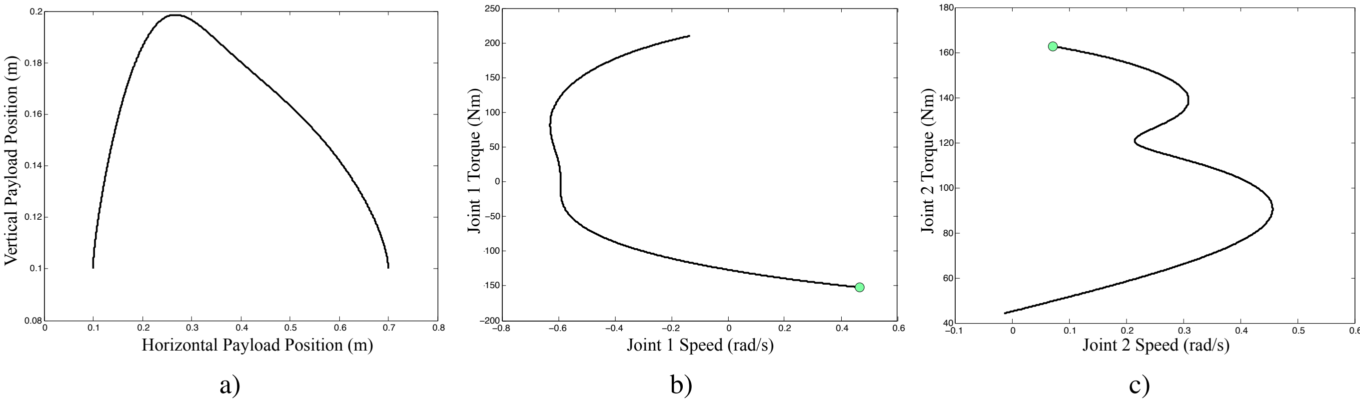


Figure 2: Optimal trajectories for nominal plant design: a) Payload trajectory b) Joint 1 and c) Joint 2 torque-speed trajectories.

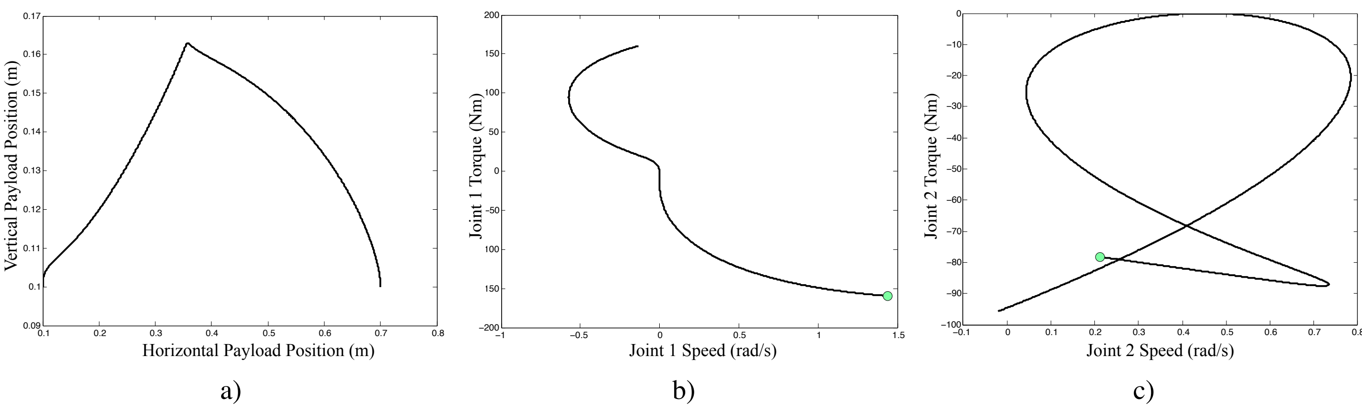


Figure 3: Task A co-design results: a) Payload trajectory b) Joint 1 and c) Joint 2 torque-speed trajectories.

PLCD results, which are later shown to offer similar performance at significantly lower cost. The Task B co-design result is:

$$\mathbf{x_p} = [1.69, 0.947, 2.46, 0.708, 28.2, 29.6]^{\mathrm{T}}$$

$$\mathbf{x_c} = [0.499, 1.46, -0.593, -0.125]^{\mathrm{T}},$$

and $E = 1.56 \times 10^{-4}$ Joules (essentially zero). Near-zero energy consumption is possible even when the payload is moved vertically 0.8 m because the mass center of the manipulator–payload system is actually lower at the end of Task B than at the beginning, owing to a well-designed counterbalance mechanism.

Figure 4 illustrates the trajectories of the Task B co-design result. As with Task A co-design, the torque-speed trajectories are always either in the forward or reverse braking quadrants. Neither joint actuator reaches the bound of 160 Nm; it would be possible to reduce actuator size here. The approximate plant modification cost, based on mass change and given by Eqn. (4), for Task B co-design is 121 kg.

3.2 Sensitivity Analysis

Normally sensitivity analysis would be performed for PLCD using the Task A plant design to perform Task B (with optimal trajectories and control). Since the original design is kinematically incapable of performing Task B, an alternate approach must be used. We do not know which requirements ($\mathbf{g}_r(\mathbf{p}) \leq \mathbf{0}$) are violated for Task B because it cannot be simulated. Instead we obtain the sensitivity of all performance metrics for Task A as an approximation to gauge which design variables have the greatest impact on improving the system design. For greater accuracy, once a design that is kinematically feasible for Task B is obtained, a second sensitivity analysis could be performed using the usual approach of simulating Task B and looking at the requirements violated for Task B.

As discussed earlier, a requirement constraint can be formed for the design objective (energy) to ensure adequate performance since in the PLCD problem cost of plant modification is used as the objective. After computing the sensitivities of energy con-

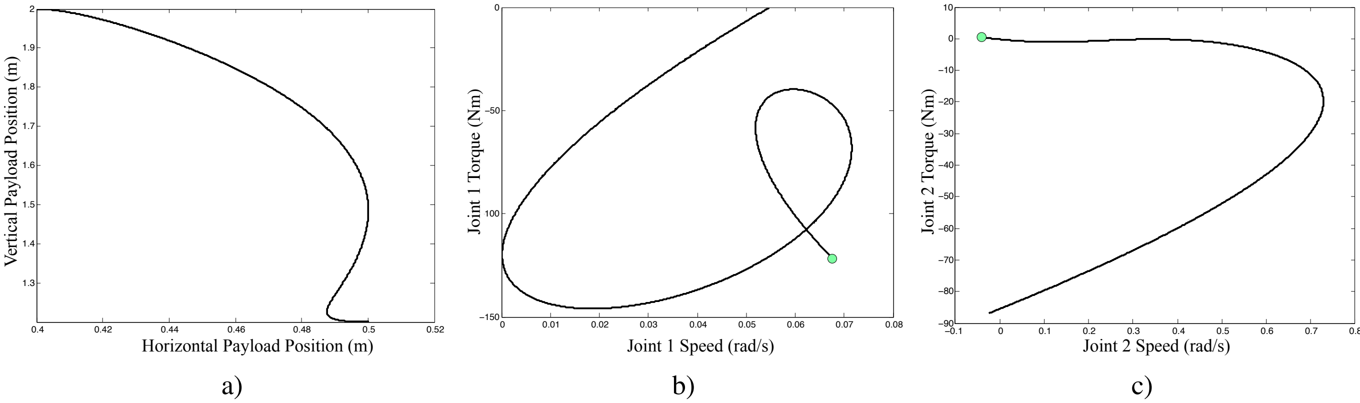


Figure 4: Task B co-design results: a) Payload trajectory b) Joint 1 and c) Joint 2 torque-speed trajectories.

sumption and the torque constraints with respect to the complete plant design vector (using optimal trajectories for each calculation), the approximate cost model based on mass was combined with these values to compute the cost Jacobian (see Eqn. (2)). Here the magnitude of each component is given:

$$\mathbf{J_c} = \begin{bmatrix} \frac{\partial E(\mathbf{p})}{\partial c_1} & \frac{\partial E(\mathbf{p})}{\partial c_2} & \frac{\partial E(\mathbf{p})}{\partial c_3} & \frac{\partial E(\mathbf{p})}{\partial c_4} & \frac{\partial E(\mathbf{p})}{\partial c_5} & \frac{\partial E(\mathbf{p})}{\partial c_6} \\ \frac{\partial g_{r1}(\mathbf{p})}{\partial c_1} & \frac{\partial g_{r1}(\mathbf{p})}{\partial c_2} & \frac{\partial g_{r1}(\mathbf{p})}{\partial c_3} & \frac{\partial g_{r1}(\mathbf{p})}{\partial c_4} & \frac{\partial g_{r1}(\mathbf{p})}{\partial c_5} & \frac{\partial g_{r1}(\mathbf{p})}{\partial c_6} \\ \frac{\partial g_{r2}(\mathbf{p})}{\partial c_1} & \frac{\partial g_{r2}(\mathbf{p})}{\partial c_2} & \frac{\partial g_{r2}(\mathbf{p})}{\partial c_3} & \frac{\partial g_{r2}(\mathbf{p})}{\partial c_4} & \frac{\partial g_{r2}(\mathbf{p})}{\partial c_5} & \frac{\partial g_{r2}(\mathbf{p})}{\partial c_6} \end{bmatrix} =$$

$$\begin{bmatrix} 3.26 & 0.0023 & 0.143 & 0.0034 & 0.0155 & 0.0193 \\ 0.0125 & 0.0334 & 0.0129 & 0.0007 & 0.694 & 0.0028 \\ 397 & 14.6 & 433 & 38.6 & 298 & 221 \end{bmatrix} \begin{matrix} \mathrm{J/kg} \\ \mathrm{Nm/kg} \\ \mathrm{Nm/kg} \end{matrix}$$

3.3 Limited Plant Redesign

In deciding which candidate plant modifications to include in the PLCD solution we must consider kinematic feasibility. The minimum total link length ($\ell_1 + \ell_2$) must be 2.04 m for $\mathbf{p}_f$ to be reachable; at least one of the link lengths must be included in the set of candidate plant modifications. It is clear from column 1 of $\mathbf{J_c}$ that ℓ_1 is most influential. ℓ_{c1}, m_{c1}, and m_{c2}, corresponding to columns 3, 5, and 6, were also selected to form the vector of candidate plant design changes:

$$\mathbf{x_p} = [\ell_1, \ell_{c1}, m_{c1}, m_{c2}]^{\mathrm{T}}.$$

The other two design variables were held fixed at the Task A optimal design values. This selection based on $\mathbf{J_c}$ was subjective; development of more formal selection techniques, including normalization and ranking of candidate modifications, is a subject for future work.

Two approaches were taken here for limited plant redesign. First, the original co-design problem given in Eqn. (5) was solved with respect to the limited plant design vector. This is not a PLCD solution since plant modification cost was not minimized, but illustrates instead a lower bound on energy consumption for the limited plant redesign. In the second approach the PLCD optimization problem given in Eqn. (3) was solved with respect to the limited design vector ($[\ell_1, \ell_{c1}, m_{c1}, m_{c2}]^{\mathrm{T}}$) to minimize plant modification cost. An upper bound on energy consumption was included: $E \leq 0.001$ Joules. The solutions of these two limited redesign problems are two points on a Pareto set that illustrates the tradeoff between modification cost and energy consumption; the solution to the first problem represents the limited redesign that minimizes energy without regard to cost, while the solution to the second problem minimizes cost with stringent limits on energy consumption. Additional Pareto-optimal designs were obtained by relaxing the energy bound to further reduce cost (Fig. 5). This tradeoff information could be used to determine whether requirement relaxation may be worth the cost savings, as well as to demonstrate the importance of making plant modification in achieving requirements for the new system application.

Figure 6 illustrates the trajectories for the first solution that minimizes energy; this solution corresponds to the upper left point on the Pareto set in Fig. 5. As with other energy-minimizing results the torque-speed trajectories remain within the forward and reverse braking quadrants. The torque at joint 1 does hit the bound of 160 Nm. The corresponding system design is:

$$\mathbf{x_p} = [1.61, 0.711, 0.126, 0.885, 15.7, 19.6]^{\mathrm{T}}$$

$$\mathbf{x_c} = [0.565, 1.48, -0.595, -0.173]^{\mathrm{T}},$$

Observe that the second and fourth plant design variables are at the same values as in the Task A optimal design — they were held fixed for the limited redesign. The energy consumption here was 7.71×10^{-4} Joules, a small increase from the Task B co-design result, but still essentially zero. The cost of these modifications

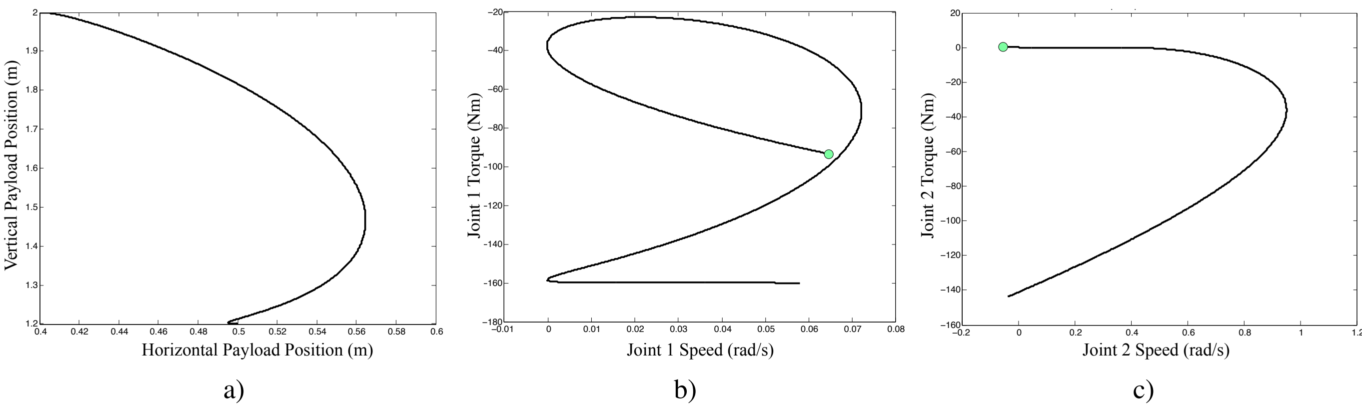


Figure 6: Task B limited redesign results (min E): a) Payload trajectory b) Joint 1 and c) Joint 2 torque-speed trajectories.

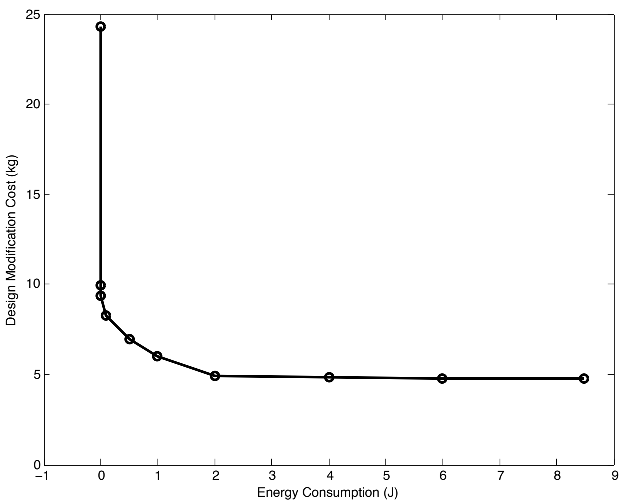


Figure 5: Pareto set illustrating the tradeoff between energy consumption and plant redesign cost.

using the approximate cost metric is 24.3 kg, significantly less than the cost of the Task B co-design result (121 kg).

In the second solution approach we demonstrate the value of PLCD by showing that plant modification cost can be reduced significantly with a relatively small increase in energy consumption. In this PLCD problem energy consumption was limited to 0.001 J, and Eqn. (3) was solved to minimize modification cost. The minimum cost was 9.95 kg, 92% lower than the Task B co-design result and 59% lower than the limited redesign result. The corresponding system design for Task B PLCD is:

$$\mathbf{x_p} = [1.56, 0.711, 0.363, 0.885, 3.89, 20.7]^{\mathrm{T}}$$

$$\mathbf{x_c} = [0.576, 1.51, -0.522, -0.288]^{\mathrm{T}},$$

Table 1: Summary of robotic manipulator design results.

	Energy Consumption	Modification Cost
Task A nominal $\mathbf{x_p}$, opt. $\mathbf{x_c}$	27.6 J	—
Task A co-design (baseline)	5.86×10^{-3} J	—
Task B co-design	1.56×10^{-4} J	121 kg
Task B redesign (min E)	7.71×10^{-4} J	24.3 kg
Task B PLCD ($E \leq 0.001$ J)	0.001 J	9.95 kg
Task B PLCD ($E \leq 2.0$ J)	2 J	4.95 kg

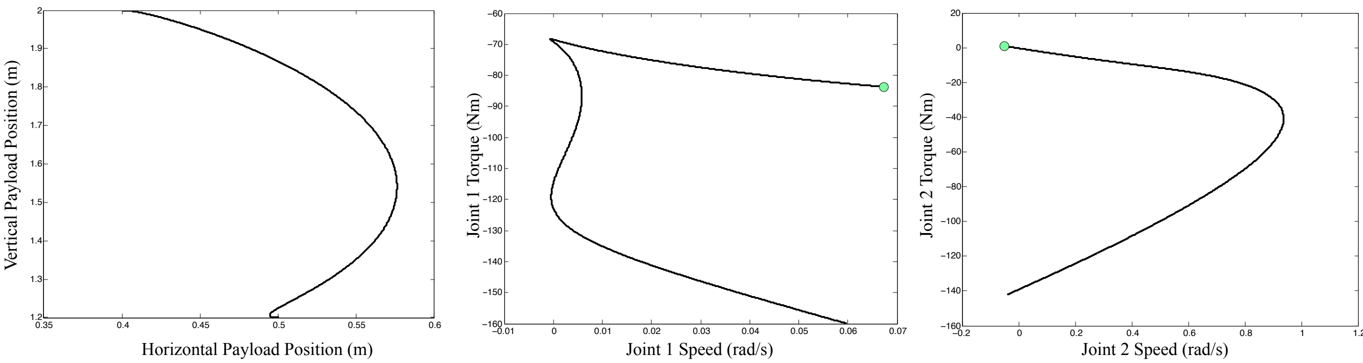
Note that the two counterbalance masses were driven back to their original Task A optimal values of 3.89 and 20.7 kg. This is due to using mass change as an approximate cost metric and the fact that most of the system mass is located in the counterbalance masses. The desired counterbalance effect can be achieved without changing counterbalance masses by adjusting link geometry with relatively little mass penalty.

Figure 7 pictures the trajectories for the Task B PLCD result where $E \leq 0.001$. Energy consumption is still very near zero. The manipulator operates almost exclusively in the forward and reverse braking quadrants, as with the other co-design results. In Figs. 4c, 6c, and 7c, note that the joint two actuator does spend a significant portion of the simulation near zero, indicating that passive dynamics are utilized partially in these designs.

Table 1 summarizes the design results from the manipulator case study, illustrating the effectiveness of the PLCD methodology for reducing redesign cost while maintaining superior performance.

4 Discussion

Plant-limited co-design (PLCD) is a recently introduced design methodology for repurposing mechatronic systems where



a) b) c)

Figure 7: Task B PLCD results ($E \leq 0.001$ J): a) Payload trajectory b) Joint 1 and c) Joint 2 torque-speed trajectories.

limited plant modifications are needed to meet requirements. This challenging task arises frequently in practice, and engineers often struggle to adapt a system to new applications through control design changes alone. PLCD methodology provides a systematic approach for identifying influential candidate design variables and minimizing the cost of physical system design changes required to meet the new application requirements. PLCD results provide quantitative justification for considering plant design modifications, and cost–performance trade-off information can be obtained using multi-objective optimization. Applying co-design methodology to a ground-up system development project is challenging, and may be impractical in many current organizations. PLCD can be viewed as an intermediate step toward adopting co-design; it requires a lower level of commitment and investment, and solves a pressing nascent problem for firms that develop mechatronic systems.

PLCD was applied to the repurposing of a counterbalanced robotic manipulator. An initial design for the original task was developed using co-design, which successfully exploited passive system dynamics to reduce energy consumption to near zero. A limited set of plant modifications was identified using sensitivity analysis, and the resulting system design cost significantly less than a full system redesign while maintaining near-zero energy consumption. The trajectories used here were quintic, placing limitations on dynamic performance. Exploring true upper bounds for control design optimization and co-design performance could be explored using unstructured open-loop control design methods such as direct transcription [43].

There are several opportunities for expanding our knowledge regarding the PLCD design methodology. More sophisticated approaches to candidate plant modification selection should be explored, as well as more detailed cost models (both for sensitivity calculation and for PLCD cost minimization). More sophisticated plant design and cost models are under development. Improved cost models would introduce discontinuities into the cost function; zero change results in zero cost, but any finite change, no matter how small, will result in finite modification cost. So far only continuous, high-level plant design changes have been considered. More detailed plant changes present interesting modeling issues. Topological changes, such as adding or removing components, introduce new challenges for co-design solution techniques.

5 Conclusion

In summary, initial PLCD investigations are promising. This methodology has been applied successfully to multiple nonlinear mechatronic systems, and has demonstrated its utility for identifying minimal-cost plant modifications while meeting demanding system requirements. While PLCD has been applied only to mechatronic systems so far, it could be expanded into a more general strategy for strategic redesign of a broader variety of system types [44], increasing the scope of its applicability and importance.

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