Engineering System Co-Design with Limited Plant Redesign

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Modern engineering systems often are impractical to design from the ground up. New designs typically are modifications of earlier systems (and frequently only small perturbations of them). This limited redesign approach is becoming increasingly important as the scale and complexity of engineering systems increases. One significant application of limited redesign is the repurposing of mechatronic systems. Often engineers seek to meet the needs of a new application by redesigning only the control system for a mechatronic device. While in many cases this approach is successful, control design changes alone may not always be sufficient. If control redesign alone is inadequate, physical system (plant) design changes should be investigated. Complete plant redesign cost may be prohibitive, and usually is unnecessary. A limited set of plant design changes should be identified that enable system requirement satisfaction at the lowest cost. Here we present a formal integrated approach for limited redesign of mechatronic systems. Candidate plant modifications are identified using sensitivity analysis, and then an optimization problem is solved that minimizes the cost of system redesign while satisfying requirements. This formal methodology for plant limited co-design (PLCD) is demonstrated using a robotic manipulator design problem. First the manipulator is designed in a way that exploits passive dynamics to minimize energy consumption for a specific task. Afterward a new task is introduced that cannot be performed successfully through control changes alone. Limited plant changes are identified, and the PLCD result for this new task is compared to a full system redesign. The PLCD result costs significantly less than the full redesign with a small performance penalty. Parametric studies illustrate the tradeoff between redesign cost and performance, and it is shown that the proposed sensitivity analysis results in the lowest cost limited redesign.

I. Introduction

Engineering systems often incorporate active control systems to govern dynamic behavior. The design of a physical system and the design of its control system are interdependent activities.[1] Conventional sequential design approaches[2–4] usually produce suboptimal results.[5] A class of design methodologies, known as co-design, account for physical system and control system design coupling and produce system-optimal results. Existing co-design strategies are intended for cases where the physical system does not already exist and the system designer has complete freedom to specify both physical and control system design. In practice, however, engineers are often faced with design problems where the physical system (plant) has already been manufactured; in this case the design objective is either to design a control system that enables the plant to be used for a new purpose, or to improve performance for its original purpose. If system performance requirements cannot be met via control system design alone, limited plant modifications should be explored to identify efficient plant design changes that enable requirement satisfaction. Modifying already manufactured physical systems is costly, so minimizing these modifications is desirable. This can be addressed in a systematic manner through plant-limited co-design (PLCD), which is a design methodology that produces system-optimal designs with minimum cost plant modifications while satisfying performance requirements for new system applications.

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The following sections outline and demonstrate a proposed methodology for PLCD. Particular attention is paid to the special challenges of developing models suitable for exploring physical system design changes. A sensitivity-based approach is employed to limit the scope of the plant design problem and the associated system model. A two-link robotic manipulator design problem is then used to demonstrate the application of PLCD to mechatronic systems. An initial manipulator design is obtained that minimizes energy consumption for a specific pick-and-place task using co-design. This initial system design capitalizes on passive system dynamics to improve energy efficiency. A second task is introduced that cannot be performed using the first system design. The manipulator is then redesigned using PLCD, and it is shown that a limited redesign can achieve energy efficiency performance comparable to a full system redesign at significantly reduced cost. PLCD is demonstrated to be an effective approach for solving an important class of mechatronic design problems that have not yet been studied in a formal way.

The work presented here is part of a broader effort to enhance our ability to redesign or reconfigure engineering systems, which is becoming increasingly important as the scale and complexity of engineering systems increases.[6,7] Completely redesigning and reimplementing large-scale engineering systems as needs evolve is impractical, especially in the case of systems-of-systems, such as transportation or energy systems.[8] Engineers must learn to manage the persistence of legacy system components that are too costly to replace while strategically modifying other system elements to achieve the desired functionality and performance in the most cost effective way.[9] Formal methods for strategic limited redesign are thus emerging as a critical segment of engineering system design, and an especially important aspect of these methods involve identification of candidate system modifications. In this article we focus on one specific type of strategic redesign: modification of mechatronic systems to meet new needs at minimal plant modification cost.

II. Plant-Limited Co-Design

Plant-Limited Co-Design is a new class of co-design methods where limitations on plant design modifications are accounted for explicitly. PLCD problems arise when engineers seek to re-purpose existing systems for new applications. For example, an existing directional antenna designed originally for maintaining a line-of-sight radio connection between a ship and aircraft might be modified to maintain a connection between a land vehicle and aircraft. The vehicle dynamics are significantly different between these two situations, and the original system may not meet performance requirements for the new system application. Suppose after exhaustive control system analysis engineers conclude that system performance requirements for the new application cannot be met through control design changes alone. Designing a new physical system from scratch would be a familiar yet expensive solution to this problem. An alternative strategy involves exploring limited changes to the existing plant design to determine if system performance requirements can be met at a reasonable expense. This strategy requires an integrated design approach where limited plant redesign is considered simultaneously with control system design. Otherwise, we would be unable to identify which elements of a physical system we could modify to meet our new requirements at minimum cost.

This article presents one possible method for solving the PLCD problem based on optimization. It is a multi-disciplinary design optimization approach with two disciplines: physical system design and control design.[10,11] Other approaches are possible and opportunities for future work in this emerging area of mechatronic system design are identified in this article.

III. PLCD Solution Process

Here we propose a formal optimization-based approach to solving PLCD problems. The following is an outline of the solution process:

1. Identify candidate plant modifications
2. Develop a system model with independent variables in the selected plant and control design spaces
3. Formulate and solve the PLCD optimization problem
4. Verify result and repeat if further improvement can be made via alternate plant modifications

In the first step we analyze the system to determine which aspects of the plant we should modify. Here we propose a sensitivity-based approach to identify plant modifications likely to have significant impact on system performance. Once this is done we can develop a model of the system that provides sufficient flexibility for exploring the selected candidate plant and control design modifications. With this model we are ready to formulate and solve the PLCD optimization problem. This article presents two approaches for solving the PLCD optimization problem. In the first we seek to minimize changes to the plant while meeting system performance constraints. In the second approach we explore the tradeoff between plant modification expense and system performance improvement using a multi-objective formulation. After we arrive at a solution we can verify whether further improvement can be achieved by altering the set of candidate plant modifications. If this is the case, the process can be repeated. The following sections describe each of these steps in detail.

III.A. Plant Modification Analysis

In solving a PLCD problem engineers may consider several different types of plant design changes, such as actuator choice, component replacement or modification, component removal or relocation, addition of new components, or other topological changes. These modifications can have a significant effect on dynamic properties. A primary objective in solving PLCD problems is to identify efficient plant modifications that enable requirement satisfaction with minimal expense and effort. Narrowing down the set of candidate plant changes eases both modeling and optimization challenges.

Here we present a first-order approach for analyzing the link between plant characteristics and system performance. This analysis may be used to select a set of candidate plant changes that form the basis of the system redesign model and PLCD optimization problem. This sensitivity-based approach applies only to candidate plant changes that are continuous in nature. Future work will address discrete modifications.

Sensitivity analysis requires a simplified system model, such as a model suitable for control system design. A model that incorporates independent plant design variables, such as geometric dimensions, is not needed at this stage. Rather, the system model may be expressed in terms of parameters that quantify plant characteristics (denoted $\mathbf{p}$ here), such as inertia values or damping rates, which is typical of models developed for control design. A simplified model may have been used for control system design when the original system was developed, and may be available for repurposing efforts.

Consider the set of system performance requirements for the new task posed in negative null form: $\mathbf{g}_r(\mathbf{p}) \leq \mathbf{0}$, and a subset of these requirements that the original system was unable to meet: $\bar{\mathbf{g}}_r(\mathbf{p}) \leq \mathbf{0}$. The sensitivity of $\bar{\mathbf{g}}_r(\mathbf{p})$ with respect to plant characteristics $\mathbf{p}$ is used to rank and select candidate plant modifications. The rationale here is to identify elements of plant design that can influence system performance as efficiently as possible.

The first sensitivity approach involves the derivatives of the violated requirements for the second task with respect to plant parameters, i.e.:

$$\frac{\partial \bar{g}_{ri}(\mathbf{p})}{\partial p_j}, \quad i = 1, 2, \ldots, n_r, \quad i = 1, 2, \ldots, n_p \tag{1}$$

where n_r is the number of violated constraints and n_p is the number of model parameters. These terms form the model parameter Jacobian $\mathbf{J}_p$, which can be used to identify the most influential parameters. A large value of $\partial \bar{g}_{ri}(\mathbf{p})/\partial p_j$ indicates that requirement i is influenced significantly by small changes in p_j. After a set of influential model parameters $\bar{\mathbf{p}}$ is identified based on $\mathbf{J}_p$, a corresponding set of candidate plant changes $\mathbf{x_p}$ that influence $\bar{\mathbf{p}}$ must be established. This link may be difficult to establish and requires change propagation analysis.[12] The system model can then be expanded based on $\mathbf{x_p}$, as described in the next step. Model expansion often requires a significant investment, so narrowing down the set of candidate plant stages here is a crucial task.

A more sophisticated analysis for selecting $\mathbf{x_p}$ involves the sensitivity of violated requirements with respect to cost:

$$\left(\frac{\partial \bar{g}_{ri}(\mathbf{p})}{\partial p_j}\right)\left(\frac{\partial C(\mathbf{p})}{\partial p_j}\right)^{-1} = \frac{\partial \bar{g}_{ri}(\mathbf{p})}{\partial c_j}, \quad i = 1, 2, \ldots, n_r, \quad j = 1, 2, \ldots, n_p \tag{2}$$

where $C(\mathbf{p})$ approximates the cost of changing model parameters, and c_j represents the cost of changing model parameter p_j. This cost model could be based on correlation between cost and these parameters on similar existing systems.[13] The $\partial \bar{g}_{ri}(\mathbf{p})/\partial c_j$ terms form the cost Jacobian $\mathbf{J}_c$, which can be used to assess more accurately which plant modifications could produce the desired performance improvements most economically.

While this step is presented as a component of the larger PLCD process, it may be used alone in conjunction with conventional design methods. For example, once $\mathbf{x_p}$ is identified using sensitivity analysis, engineers can proceed using conventional design methods to modify the plant and then the control system in a sequential manner. In some cases this approach may be sufficient, and the relatively small modeling and analysis investment is appealing. Mattila and Virvalo, for example, applied an informal version of PLCD step 1 by identifying critical elements of a hydraulic manipulator and redesigning them using conventional techniques to achieve significant energy savings.[14] Observe, however, that this sequential approach is not a co-design method; potential synergy between plant and control design cannot be exploited and the result will not be system-optimal. If this simplified approach is not successful the full co-design process described above should be used. In addition, if the full co-design process fails to identify a new design that satisfies requirements based on a particular $\mathbf{x_p}$, the criteria for selecting $\bar{\mathbf{p}}$ should be relaxed to increase the dimension of $\mathbf{x_p}$.

III.B. System Model Development

Solving the PLCD optimization problem described in step 3 requires a system model that has as independent variables the candidate plant modifications $\mathbf{x_p}$ identified in step 1. Models that predict accurately the results of physical system design changes are challenging to develop, often requiring significant resource investment. Developing a system model that accommodates all possible plant changes is impractical and unnecessary for realistic PLCD problems; reducing the size of $\mathbf{x_p}$ is important for curbing model development expense and easing optimization solution difficulty.

One possible modeling approach is to augment the existing control design oriented model used in step 1 with specialized modeling tools to predict model parameter values. The objective is to form a model $\mathbf{p} = \mathbf{a}(\mathbf{x_p})$, where $\mathbf{a}(\mathbf{x_p})$ is an analysis function that computes model parameters as a function of independent plant design variables using computer aided engineering tools such as finite element analysis. This unidirectional system model structure is described in more detail by Allison and Nazari[10] and Frischknecht et al.[15] Bidirectional coupling between plant and control design may still be captured through design constraint dependence on control design variables $\mathbf{x_c}$.

III.C. Plant Modification Minimization

Once candidate plant modifications $\mathbf{x_p}$ have been identified and a suitably flexible system model has been developed, an optimization-based approach may be used to identify the minimum plant modification required to meet system requirements. Consider the following PLCD optimization problem:

$$\begin{aligned} \min_{\mathbf{x}=[\mathbf{x_p},\mathbf{x_c}]} \quad & \phi(\mathbf{x}) \\ \text{s.t.} \quad & \mathbf{g_p}(\mathbf{x}) \leq 0 \\ & \mathbf{g_r}(\mathbf{x}) \leq 0 \end{aligned} \tag{3}$$

where $\phi(\mathbf{x})$ is a measure of deviation from the original plant design, $\mathbf{g_p}(\mathbf{x})$ are plant design constraints, and $\mathbf{g_r}(\mathbf{x})$ are system performance requirements formulated as inequality constraints. Both sets of constraints are assumed to be given here. Note that $\mathbf{g_r}(\mathbf{x})$ represents the full set of system requirements, not just those used in step 1 for identifying candidate plant modifications. While omitted from this formulation, control design constraints such as stability may be included here. The optimization variable $\mathbf{x_p}$ parametrizes the candidate plant modifications, and $\mathbf{x_c}$ are the control design variables. The solution to this problem is a system design that meets performance requirements while requiring minimal modifications to the physical system design.

The choice of metric $\phi(\mathbf{x})$ has significant impact on the resulting design solution. A weighted norm ($\phi(\mathbf{x}) = \|\mathbf{w} \circ (\mathbf{x_0} - \mathbf{x_p})\|$) that accounts for varying difficulty in different plant modifications as well as the magnitude of the components of $\mathbf{x_p}$ might be used as a simplified metric, or ideally a more sophisticated (nonlinear) cost function that estimates the expense of a given plant modification may be used. In the robotic manipulator case study here cost of plant changes are approximated using change in mass. With this simplified metric we estimate the cost of limited plant redesign to the manufacturer. A more complete study would include full lifecycle cost, including use (such as energy and maintenance costs) and end-of-life costs.

III.D. Multi-objective Optimization

If the performance requirements are flexible the engineer may wish to explore the tradeoff between performance and plant redesign cost. This tradeoff information is especially useful if the design objective is to improve system performance at a reasonable cost, and can be obtained via solution of a multi-objective optimization problem:

$$\begin{aligned} \min_{\mathbf{x}=[\mathbf{x_p},\mathbf{x_c}]} \quad & \{\phi(\mathbf{x}), \psi(\mathbf{g_r}(\mathbf{x}))\} \\ \text{s.t.} \quad & \mathbf{g_p}(\mathbf{x}) \leq 0. \end{aligned} \tag{4}$$

Here we seek to minimize simultaneously the scalar cost metric described above $\phi(\mathbf{x})$, as well as $\psi(\mathbf{g_r}(\mathbf{x}))$, the maximum normalized performance violation. The resulting Pareto set provides insight into the tradeoff between cost and requirement satisfaction, which is useful in determining whether some relaxation of performance requirements is worth the associated cost savings.

The Pareto set also illustrates the predicted minimum performance degradation if no plant design changes are made, i.e., $\|\mathbf{x_0} - \mathbf{x_p}\| = 0$. This is useful quantitative evidence the engineer can use in supporting the case for limited plant design changes. If the cost of not meeting requirements exceeds the cost of plant changes required to satisfy requirements, then proceeding with a limited plant redesign is justified.

IV. Plant-Limited Co-Design of a Robotic Manipulator

Robotic manipulators are used extensively in manufacturing; manipulator energy efficiency and dynamic performance are important economic and environmental considerations.[16–18] The PLCD example presented here involves a two-link planar manipulator that is designed to perform a specific pick-and-place task (Task A) with minimal energy consumption while complying with joint actuator torque and link deflection constraints. This baseline design is obtained using co-design, and represents the existing mechatronic system an engineer seeks to modify to perform a new task (Task B). It is shown that the baseline design is incapable of meeting Task B requirements through control design changes alone. Sensitivity analysis is used to identify a limited set of plant modifications, and then the PLCD problem is solved to identify a limited plant redesign that can perform Task B while complying with requirements. PLCD is also compared to full system redesign.

Figure 1a illustrates the manipulator configuration. Position is specified by the two joint angles θ_1 and θ_2 (in the position shown $\theta_2 < 0$). Each link has a constant annular cross section with radius r_i and wall thickness t_i (Fig. 1b) and is constructed of 7075 T6 aluminum. Figure 1c illustrates Task A initial and final conditions. The manipulator is to lift a 20 kg payload from the initial position $\mathbf{p}_0$ with initial velocity $\mathbf{v}_0$, and $t_f = 2.0$ seconds later place the payload at $\mathbf{p}_f$ with final velocity $\mathbf{v}_f$.

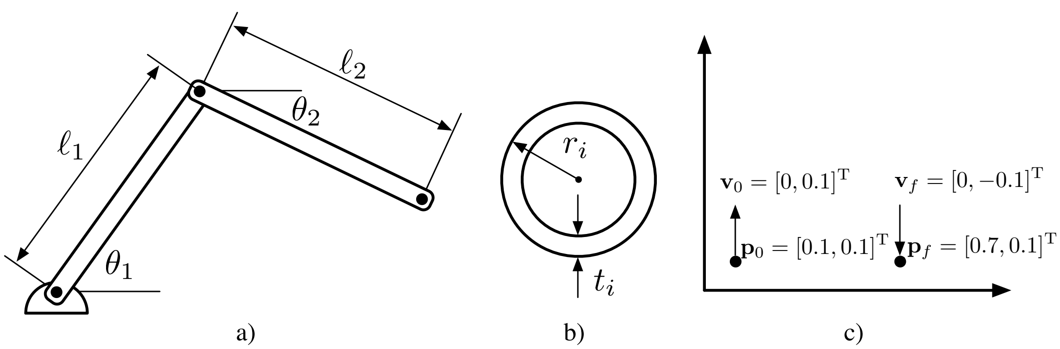


Figure 1. a) Two-link planar manipulator. b) Section view of link i. c) Task A initial and final conditions.

The original system is designed to perform Task A with minimal energy based on the following co-design formulation:

$$\begin{aligned} \min_{\mathbf{x}=[\mathbf{x_p},\mathbf{x_c}]} \quad & E(\mathbf{x}) & \\ \text{s.t.} \quad & |\tau_{\max,i}(\mathbf{x})| \leq \tau_{\text{allow}}, & i = 1, 2 \\ & \delta_i(\mathbf{x_p}) \leq \delta_{\text{allow}}, & i = 1, 2 \end{aligned} \tag{5}$$

where $E(\mathbf{x})$ is the total mechanical energy consumed to perform the assigned task, $\tau_{\max,i}(\mathbf{x})$ is the maximum joint i torque, τ_{allow} is the allowable torque (limited by joint actuators), $\delta_i(\mathbf{x_p})$ is the maximum link deflection for a given nominal torque, and δ_{allow} is the upper deflection bound. Here we assume the actuator design is fixed, prescribing the torque bound of τ_{allow} 210 Nm. The plant design is limited to the two link lengths and the link section radius:

$$\mathbf{x_p} = [\ell_1, \ell_2, r_1, r_2]^{\mathbf{T}}.$$

The desired (quintic) trajectory for the joint angles $\mathbf{q_d}(t) = [\theta_1(t), \theta_2(t)]^{\text{T}}$ is calculated based on the initial and final positions and velocities given in Fig. 1c, as well as the position $\mathbf{p}_I$ and velocity $\mathbf{v}_I$ of an intermediate point along the path between $\mathbf{p}_0$ and $\mathbf{p}_f$. A feedback linearization approach[19] was used to track $\mathbf{q_d}(t)$. Joint torque trajectories and energy consumption computed using inverse dynamics agreed with feedback linearization, allowing omission of tracking control design variables and simplification of the problem formulation. The reduced set of control design variables include the intermediate position and velocity values required to define the desired trajectory:

$$\mathbf{x_c} = [p_{I1}, p_{I2}, v_{I1}, v_{I2}]^{\mathbf{T}}.$$

The following nonlinear differential equation[19] was used to model manipulator dynamics:

$$\mathbf{M}(\mathbf{q}, \mathbf{x_p})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{x_p})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}, \mathbf{x_p}) = \boldsymbol{\tau}, \tag{6}$$

where $\mathbf{M}(\mathbf{q}, \mathbf{x_p})$ is the inertia matrix, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{x_p})$ computes the centrifugal and Coriolis terms, and $\mathbf{g}(\mathbf{q}, \mathbf{x_p})$ is the gravity vector. Note that each of these terms depend on both joint position and plant design. The joint torque vector is $\boldsymbol{\tau} = [\tau_1, \tau_2]^{\text{T}}$. In this model joint motor mass, motor rotational inertia, and electrical losses have been neglected. Energy is calculated by integrating the mechanical power at each joint, but no energy recapture is counted (i.e., no regenerative braking).

Using monotonicity analysis[20] the deflection constraint can be shown to be active, and substitution can be used to eliminate radius values from $\mathbf{x_p}$. The deflection constraint is satisfied implicitly, and the reduced dimension design vector is $\mathbf{x_p} = [\ell_1, \ell_2]^{\mathbf{T}}$. More specifically, given ℓ_i, we can calculate the minimum value of r_i that satisfies the deflection constraint. Here we assume that the nominal torques are $\boldsymbol{\tau_n} = [140, 80]^{\text{T}}$ Nm, $\delta_{\text{allow}} = 4\,\mu$m, link wall thicknesses are $t_1 = 3$ mm and $t_2 = 2$ mm, the elastic modulus is $E = 71.7$ GPa, and material density is $\rho = 2810$ kg/m^3 to calculate radius values:

$$r_i = \frac{2\tau_{ni}\ell_i^2}{3\pi E t_i \delta_{\text{allow}}} + \frac{t_i}{2} \tag{7}$$

IV.A. Task A Co-Design

In this section we present the solution of the co-design problem given in Prob. (5) for Task A, and discuss the role of passive dynamics in reducing energy consumption. The resulting design is used as a baseline system design that represents the existing system to be modified via PLCD. For comparison and to demonstrate the value of co-design, we will first consider the performance of a nominal plant design ($\mathbf{x_p} = [0.6, 0.6]^{\text{T}}$ m) that represents a design solution obtained through conventional means (i.e., plant design obtained via expert engineering intuition followed by control optimization[a]). The minimum energy trajectory for the nominal design is $\mathbf{x_c} = [0.183, -0.0836, 0.0146, 0.142]^{\text{T}}$, which results in the payload path illustrated in Fig. 2a. Figures 2b-c illustrate the torque-speed trajectories for each of the joints; the circle indicates the starting point at the initial time t_0.

Both joint actuators stay well within the bound $\tau_{\text{allow}} = 210$ Nm. The payload trajectory follows a 'falling' type motion, exploiting to some degree the passive dynamics of the baseline plant design to perform Task A using only 21.3 J of energy. The torque at joint 1 remains near zero for much of the simulation; joint 2 is more active.

Understanding and harnessing the intrinsic dynamics of a physical system[23] can help reduce control forces and energy inputs. Rather than using joint control to force the manipulator to follow a specific path, we allow it to follow the passive trajectory as much as possible, exerting relatively small control torques to guide the payload into the right position at the right time. Taking this idea to the extreme, McGeer[24] demonstrated

[a] Optimization problems throughout this article were solved using a hybrid approach where a gradient-based method (SNOPT[21]) is applied after a gradient-free method (pattern search with a global search method[22]) to improve the probability of finding a global optimum.

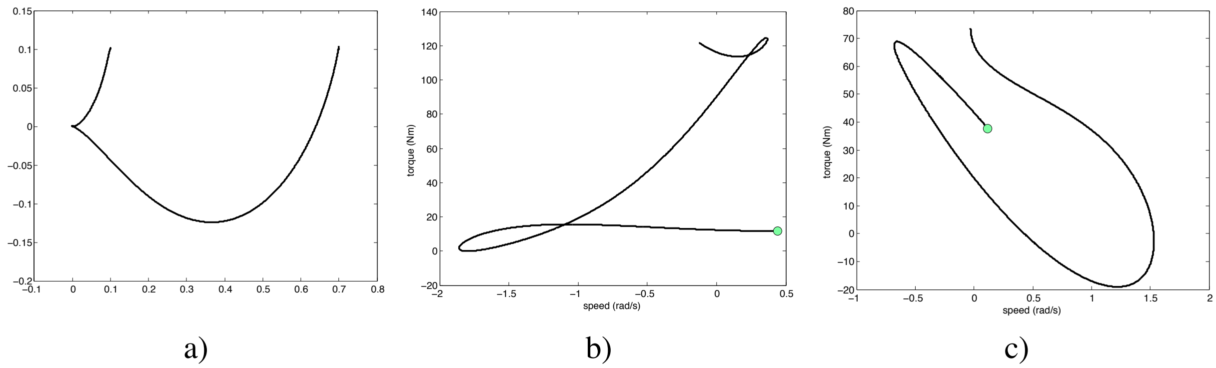


Figure 2. Nominal plant design results: a) Payload trajectory b) Joint 1 and c) Joint 2 torque-speed trajectories.

a passive walking device that required no active input — only gravitational potential energy — to maintain steady-state walking. Collins et al.[25] used simple power sources to replace gravity for passive walkers, enabling them to walk on flat ground or inclines with energy efficiency magnitudes better than conventional robotic walkers. Williamson applied this principle to a variety of other robotic systems, explaining that using passive dynamics instead of ignoring or canceling them can be particularly beneficial for specialized or repetitive tasks.[26]

Ahmadi and Buehler classify the use of passive dynamics as a biomimetic principle that can be used to reduce energy consumption in dynamic systems.[27] There is one important distinction between previous studies in passive dynamics and biological systems; the former fixes physical system design and addresses only control design, whereas physical and control system properties co-evolve in biological systems. Here we take robotic design a step closer to the elegance of biological systems by designing physical and control systems together in a way that exploits synergy between these design disciplines. The nominal design represented in Fig. 2 is efficient, but can be improved by tailoring the physical system to the requirements of Task A. The simultaneous solution to Problem (5) is system-optimal, the trajectories of which are illustrated in Fig. 3.

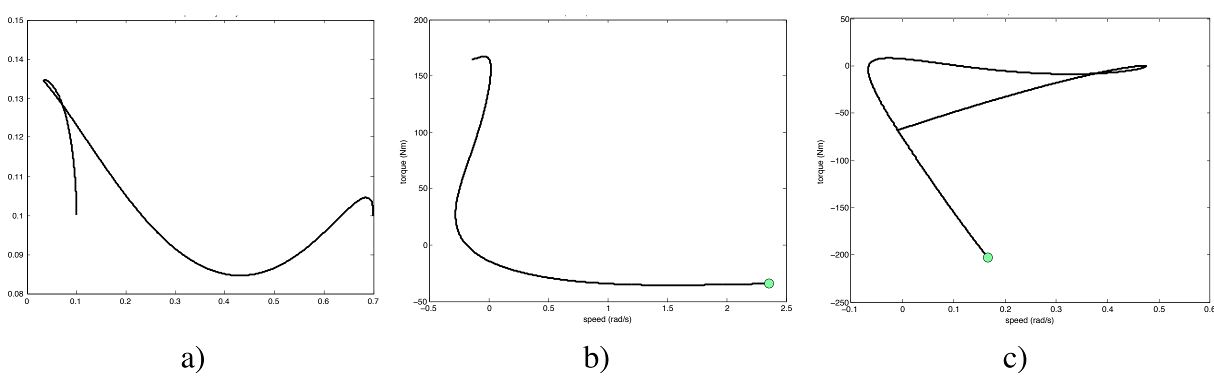


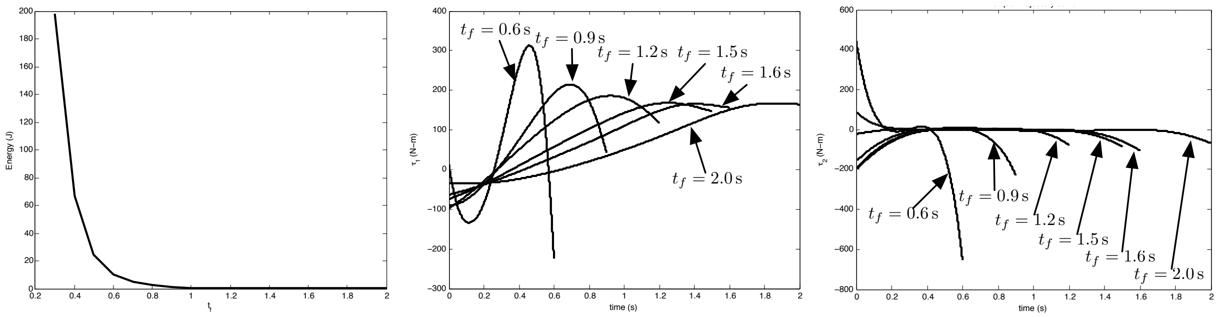
Figure 3. Task A system-optimal design: a) Payload trajectory b) Joint 1 torque-speed trajectory c) Joint 2 torque-speed trajectory.

The projectile takes a fundamentally different path with the new design; joint torques remain close to zero for much of the simulation. The total energy consumption is a remarkably low 0.0272 Joules, and the optimal design is:

$$\mathbf{x_p} = [1.77, 1.63]^{\mathrm{T}}, \quad \mathbf{x_c} = [0.113, 0.121, 0.0503, -0.437]^{\mathrm{T}}$$

The links more than doubled in length, a counterintuitive result. The system mass is higher, but energy consumption decreased. In this case the center of mass location and kinematics were ideal for Task A; customized passive dynamics enabled task completion with very little control effort. While τ_2 is at its bound at t_0, it is much smaller afterwards. Also note that for Task A links must have similar lengths so that $\mathbf{p}_0$ is reachable.

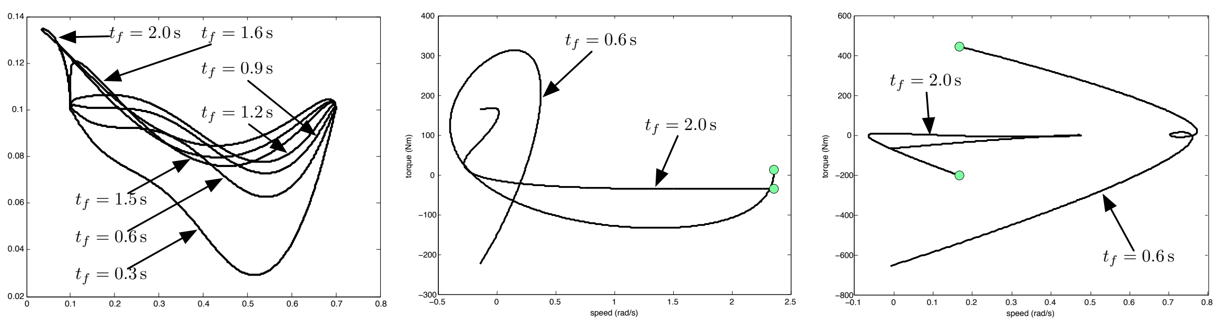
A parametric study was performed to explore the influence of task time t_f on system performance. Holding the plant design fixed at the system optimal value of $\mathbf{x_p} = [1.77, 1.63]^{\mathrm{T}}$, task time was varied from 0.3 to 2.0 seconds and the optimal trajectory was computed for each task time (Figs. 4 and 5).



a) b) c)

Figure 4. Task A parametric study: a) Energy consumption as a function of task time b) Joint 1 and c) Joint 2 actuator torques for various task times.

Figure 4a reveals the dependence of energy consumption on task time; as expected energy consumption and t_f are inversely related. Energy consumption is nearly constant between $t_f = 1.0$ and 2.0 seconds, indicating that passive dynamics dominate and very little control intervention is required in this range. Below this range more energy is required because the manipulator must move faster than passive dynamics allow. Figures 4b and 4c illustrate the joint torque trajectories for a range of task time values. Trajectories for $t_f < 0.6$ seconds are omitted due to large magnitudes that would obscure the other trajectories.



a) b) c)

Figure 5. Task A parametric study: a) Payload trajectory as a function of task time b) Joint 1 and c) Joint 2 torque-speed trajectories for $t_f = 0.6$ sec and $t_f = 2.0$ sec.

Figure 5a illustrates the projectile trajectories for a range of task times. Trajectories undergo a fundamental change in shape as task time dips below one second. Above this value the payload is first lifted in a way that exploits passive dynamics. Below one second the passive dynamics are too slow, and the payload takes a more direct (forced) path. The torque-speed curves for $t_f = 0.6$ and $t_f = 2.0$ seconds shown in Figs 5b and 5c illustrate that longer task times (which are more aligned with passive dynamics) result in torque trajectories closer to zero for a greater portion of the task. This is one way to identify whether passive dynamics dominate system behavior.

Keep in mind that in this parametric study the plant design was held fixed. Except for $t_f = 2.0$ seconds this plant design is not system-optimal. Had we performed co-design for each t_f value the energy consumption and torques would be lower as the passive dynamics would be better matched to the task. There is still a lower task time limit for physical systems below which passive dynamics cannot be used; below this limit active control must dominate since passive dynamics are incapable of sufficiently fast motion.

IV.B. Task B Co-Design

Here we will first explore the performance of the Task A system-optimal plant design for a new task (Task B) and verify that this plant design is incapable of meeting torque requirements through control system (trajectory) design changes alone. We will then investigate the improved performance of a new system design that was obtained via co-design for Task B (i.e., design from scratch for the new task). This result reveals the best possible performance for Task B through complete system redesign, but comes at a cost that may be impractical. The next step, described in the following subsection, involves co-design with limitations on plant redesign as a strategy to reduce system modification cost.

The new task (Task B) involves significant vertical displacement, and is defined by the following boundary conditions:

$$\mathbf{p}_0 = [0.5, 1.2]^{\mathrm{T}}\,\mathrm{m}, \quad \mathbf{v}_0 = [-0.1, 0]^{\mathrm{T}}\,\mathrm{m/s}, \quad \mathbf{p}_f = [0.4, 2.0]^{\mathrm{T}}\,\mathrm{m}, \quad \mathbf{v}_f = [-0.1, 0]^{\mathrm{T}}\,\mathrm{m/s}$$

If the manipulator links were massless, the energy required to perform Task B could not be less than $m_p g h = (20\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(0.8\,\mathrm{m}) = 157$ J. Values lower than this are possible if the total mass center at the initial position was higher than the payload, or if the total mass center at the final position was lower than the payload. This highlights the one influence of plant design on potential system performance.

Holding the optimal plant design from Task A co-design fixed while optimizing the joint trajectories $\mathbf{q}(t)$ for Task B, the minimum energy consumption 115.7 J. Here we attempted to impose the torque limit of $\tau_{\mathrm{allow}} = 210$ Nm, assuming that joint actuators in the baseline design are to be reused. Unfortunately the maximum joint torques here are $\tau_{1\mathrm{max}} = 210$ Nm and $\tau_{2\mathrm{max}} = 401$ Nm, illustrated in Figs. 6b and c. This result confirms that the *Task A plant design is incapable of performing Task B while satisfying torque requirements*. It can be shown by parametrically adjusting torque limits and iteratively solving the optimal trajectory problem that Task B is feasible for the Task A design if actuators capable of 382 Nm of torque are used.

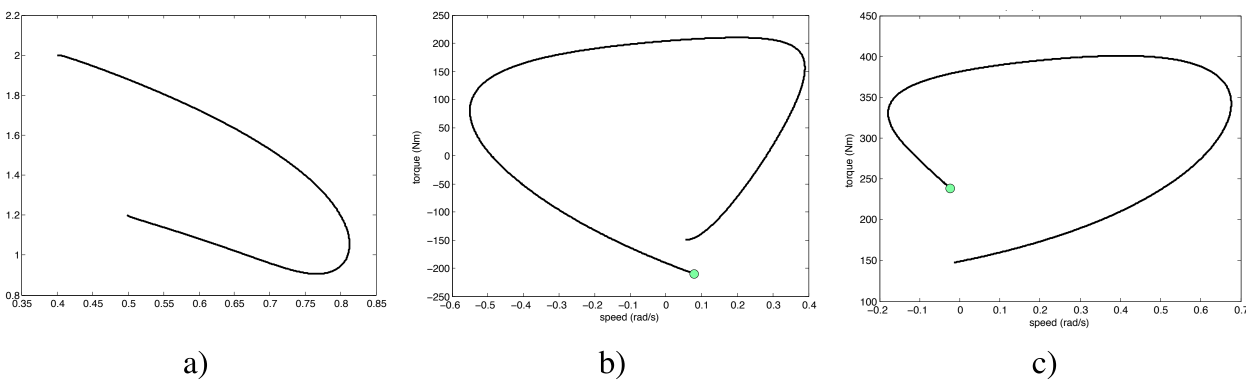


Figure 6. Task B using Task A plant design: a) Payload trajectory b) Joint 1 and c) Joint 2 torque-speed trajectories.

Observe from Fig. 6 that passive dynamics are not exploited; torque values remain far from zero. Task B performance clearly can be improved by redesigning the entire system specifically for Task B requirements, although this may be a costly option. Task B co-design is presented here as an upper performance bound for comparison with PLCD results offered in the next subsection. The minimum energy consumption obtained using co-design was 52.6 J, and the optimal system design is:

$$\mathbf{x_p} = [2.28, 1.14]^{\mathrm{T}}, \quad \mathbf{x_c} = [0.691, 1.52, -0.758, -0.0618]^{\mathrm{T}}$$

Referring to Fig. 7, the trajectory is fundamentally different. The Task A plant design was optimized early for a specific falling motion, which is evident in Fig. 6a, whereas the Task B co-design trajectory is much more efficient for Task B. The longer (and no longer similar) link lengths place the system mass center to the upper left of the payload, enabling the payload to be hoisted into position using passive dynamics to some degree. While the torque values illustrated in Figs. 7b and c are not near zero, they are significantly lower than when using the Task A plant design.

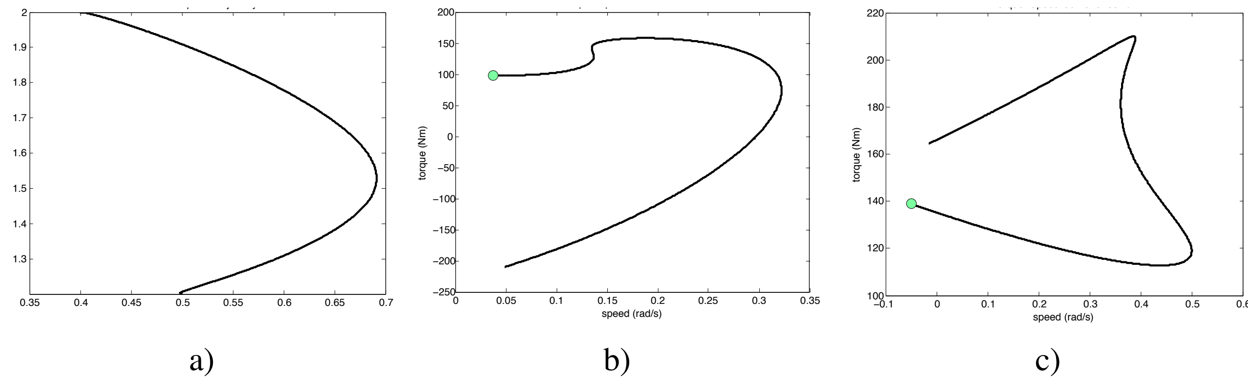


Figure 7. Task B co-design results: a) Payload trajectory b) Joint 1 and c) Joint 2 torque-speed trajectories.

IV.C. Plant-Limited Co-Design for Task B

The Task B co-design result performed well, but was costly. Here we will use change in link mass as a proxy cost function. In the Task B co-design result, link 1 increased in length by almost 30%. Referring to Eqn. (7), link mass increases faster than proportionally with link length. Here PLCD will be applied to obtain a system design for Task B that meets requirements while minimizing the cost of plant design changes.

Step 1 in the PLCD process is to identify candidate plant design changes. The plant design in this system has been simplified to two design variables, so a limited plant redesign will involve changing either ℓ_1 or ℓ_2 (a more sophisticated limited plant redesign example can be found in a companion article[28]). When attempting to use the Task A plant design to perform Task B, we discovered that the first torque constraint was active and the second was violated. We are interested in learning which of the two design variables would be most effective at reducing both maximum joint torques. This is done here using sensitivity analysis.

Both the $\partial \bar{g}_{ri}(\mathbf{p})/\partial p_j$ and $\partial C(\mathbf{p})/\partial p_j$ terms for Eqn. (2) were computed using finite differences, where $\bar{g}_{ri}(\mathbf{p})$ are the torque constraint violations and $C(\mathbf{p})$ is the change in mass from the Task A plant design. Having gone through the Task A co-design process, a complete system model that accommodates the independent design variables is available (which is normally not the case), so $\mathbf{p}$ is equivalent to $\mathbf{x_p}$. The resulting cost Jacobian is:

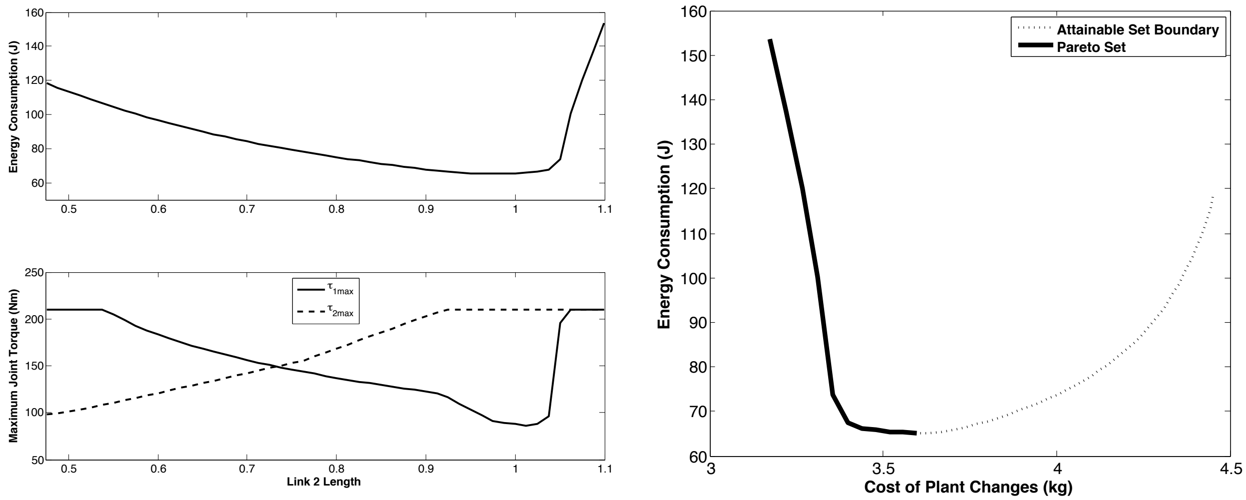
$$\mathbf{J_c} = \begin{bmatrix} \frac{\partial \bar{g}_{r1}(\mathbf{p})}{\partial c_1} & \frac{\partial \bar{g}_{r1}(\mathbf{p})}{\partial c_2} \\ \frac{\partial \bar{g}_{r2}(\mathbf{p})}{\partial c_1} & \frac{\partial \bar{g}_{r2}(\mathbf{p})}{\partial c_2} \end{bmatrix} = 10^3 \times \begin{bmatrix} 0.205 & -1.10 \\ -0.00758 & 0.0525 \end{bmatrix} \quad \text{Nm/kg}$$

The first column indicates how sensitive joint torque violation is with respect to the cost of changing ℓ_1 (approximated using mass), and the second column expresses this sensitivity for link 2. Clearly link 2 is dominant, so we will proceed with the rest of the PLCD process based on a plant redesign that consists only of ℓ_2 as the candidate plant design variable. In problems with more sophisticated plant design problems the set of candidate plant design variables would normally be larger than one.[28] Note that some candidate plant design variables may go unchanged during the optimization step (step 3) of the PLCD process, particularly if there is a large step increase of cost for any non-zero change of a plant design variable.

The second step in the PLCD process specifies the development of a more complete system model that incorporates dependence on candidate plant design variables. A complete system model is already available from Task A co-design, so this step can be skipped. Normally a model like this is not available; engineers at this stage will have only a model useful for control design and would need to put forth significant effort to develop a model that relates independent design variables to control design model parameters.

In addition to solving Problem (3) for this example, we performed a parametric study on ℓ_2 to illustrate tradeoffs involved in this PLCD problem. Figure 8a illustrates the relationship between ℓ_2 and energy consumption, as well as maximum joint torque values. This shows that when choosing ℓ_2 as the candidate plant design variable results in a large feasible domain for the PLCD problem ($0.48 \leq \ell_2 \leq 1.10$ m). In other words, not only is it possible to meet Task B requirements by varying only ℓ_2, but many options exist, providing opportunity to reduce cost further. The minimum cost design that solves Eqn. (3) is $\ell_2 = 1.10$ m. This is the design that is closest to the original length of ℓ_2 from the Task A design (1.63 m) (Fig. 8a). The (proxy) cost of changing ℓ_2 to 1.10 m is 3.17 kg. If instead we would like to find the design that minimizes energy, the result is $\ell_2 = 0.975$ m. This comes at the price of a small cost increase to 3.59 kg, but reduces energy consumption to 65.15 J.

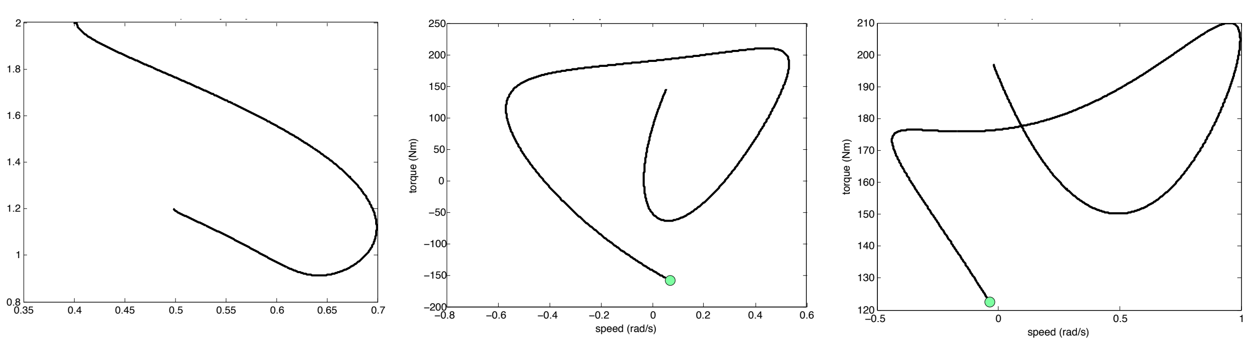
Figure 8b illustrates the tradeoff between the (proxy) cost of plant changes and energy consumption. At the upper left end of the curve is the minimum cost solution, but we can see that energy consumption can be reduced significantly with only nominal cost increases. The dashed line indicates the boundary of the attainable set that does not lie on the Pareto frontier. Note that in this case use of a complete lifecycle cost metric for $\phi(\mathbf{x_p})$ that includes both redesign and use (energy) cost is an alternative to multi-objective optimization, although tradeoff information can be useful in supporting the decision to make limited plant design changes.



a) b)

Figure 8. PLCD for Task B: a) Parametric study on ℓ_2, b) Pareto set illustrating cost–energy tradeoff.

Figure 9 illustrates the projectile and toque-speed trajectories for the minimum-cost design resulting from the Task B PLCD solution. The projectile trajectory is fairly similar to the trajectory associated with using the Task A design for Task B. Torque does not remain near zero, indicating passive dynamics do not play a large role in system response. Passive dynamics become more significant when switching to the minimum energy solution ($\ell_2 = 0.975$ m). Maximum torque for both joints in the minimum cost solution is 210 Nm, meeting Task B requirements.



a) b) c)

Figure 9. Task B PLCD Results: a) Payload trajectory b) Joint 1 and c) Joint 2 torque-speed trajectories.

Given the simplicity of plant design here, we can verify that ℓ_2 was the correct choice for limited plant redesign by investigating the outcome of choosing ℓ_1 instead. Choosing ℓ_1 does in fact result in a feasible PLCD problem, but the feasible domain is very narrow ($0.42 \leq \ell_1 \leq 0.45$ m), plant modification cost is higher (min cost is 10.0 kg), and energy consumption is much higher (354 J). In this case the sensitivity-based approach to selecting candidate plant modifications succeeded. Other more sophisticated approaches should be explored, including those that address change propagation through a system more thoroughly.[12]

V. Discussion

As engineering system scale and complexity increase, limited system redesign (as opposed to designing from scratch) is becoming more commonplace. In redesign of mechatronic systems often engineers seek to meet performance requirements for new applications through control design changes alone. While usually less expensive than physical design changes, control design changes alone may be insufficient. If control system modification is inadequate, limited physical system design changes should be investigated (complete system design may be impractical).

In this article we proposed a solution for mechatronic system repurposing that involves a limited redesign of the physical system (plant). Here a subset of plant components are selected for redesign, reducing the cost of plant changes. The limited plant and the control design changes could be made using a traditional sequential approach, or a 'co-design' approach could be used where plant and control design changes are considered simultaneously to produce a superior system-optimal design. The latter approach, detailed in this article, is the first formal methodology for plant-limited co-design, i.e, an integrated approach for solving the system repurposing problem with limited plant changes. Candidate plant design changes are identified using sensitivity analysis, and optimization is used to solve the resulting plant-limited co-design (PLCD) problem. It was shown that the sensitivity analysis did lead to the correct candidate plant modifications using a robotic manipulator design example. In this example the objective was to perform a specific manipulation task in a prescribed amount of time while minimizing energy consumption and satisfying deflection and torque constraints. This example also demonstrated the effectiveness of co-design for exploiting passive system dynamics to reduce energy consumption.

The primary contribution here is the development of a formal approach for solving mechatronic PLCD problems. This approach may be viewed as an intermediate step for industry toward full-system co-design. Engineering firms may be reluctant to embrace co-design methods for ground-up development of new mechatronic systems; PLCD, however, may be easier to adopt because of reduced model development requirements, smaller adoption investment, and rapid realization of PLCD benefits. PLCD can aid in reducing cost, energy consumption, and material usage in the development of mechatronic systems by supporting the reuse of existing systems. PLCD also has value beyond repurposing; systems that perform poorly for their original task may be improved via limited system redesign.

Formalization of a limited redesign approach for mechatronic systems establishes PLCD as a new design paradigm and generates numerous opportunities for future work. The example presented here involved a simplified physical system design; several questions still need to be addressed, including how to accommodate topological plant design changes such as adding, removing, or replacing components in addition to modifying existing components. More accurate cost modeling is needed, and the sensitivity analysis used here may not work for system design problems with a larger or more complicated model or design space. Here a very simplified control design approach was used; future work may involve detailed control design decisions, observer design, or sensor/actuator placement. This design strategy may be extensible to other systems requiring redesign besides mechatronic systems.

VI. Conclusion

In summary, PLCD is a promising new area of mechatronic design that supports the repurposing of existing systems for new applications or improving underperforming systems. It has the potential to produce significant cost, energy, and material savings, and is easier for practitioners to implement than full system co-design. Continued development of PLCD methodology and application to new case studies should be pursued to enhance our ability to design and manage increasingly complex engineering systems.

Acknowledgements

The author would like to acknowledge the insightful feedback on this work from Zhi Han of MathWorks, Inc. and Jeremy Michalek and Orkun Karabasoglu of Carnegie Mellon University.

References

[1]Fathy, H., *Combined Plant and Control Optimization: Theory, Strategies and Applications*, Ph.D. dissertation, University of Michigan, 2003.

[2]Li, Q., Zhang, W., and Chen, L., "Design for Control — a Concurrent Engineering Approach for Mechatronic Systems Design," *IEEE/ASME Transactions on Mechatronics*, Vol. 6, No. 2, 2001, pp. 161–169.

[3]Friedland, B., *Advanced Control System Design*, Prentice-Hall, 1996.

[4]Roos, F., *Towards a Methodology for Integrated Design of Mechatronic Servo Systems*, Ph.D. thesis, Royal Institute of Technology, 2007.

[5]Reyer, J., Fathy, H., Papalambros, P., and Ulsoy, A., "Comparison of Combined Embodiment Design and Control Optimization Strategies Using Optimality Conditions," *Proceedings of the ASME Design Engineering Technical Conferences*, Vol. 2, 2001, pp. 1023–1032.

[6]Siddiqi, A. and de Weck, O. L., "Modeling Methods and Conceptual Design Principles for Reconfigurable Systems," *Journal of Mechanical Design*, Vol. 130, No. 10, 2008, pp. 101102–1–15.

[7]de Weck, O., Roos, D., and Magee, C., *Complex Systems: Meeting Human Needs in a Complex Technological World*, MIT Press, Cambridge, MA, 2011.

[8]DeLaurentis, D., "Understanding Transportation as System-of-Systems Design Problem," *43rd AIAA Aerospace Sciences Meeting and Exhibit*, AIAA, 2005, pp. 1–14.

[9]Harper, S. R. and Thurston, D. L., "Incorporating Environmental Impacts in Strategic Redesign of an Engineered System," *Journal of Mechanical Design*, Vol. 130, No. 3, 2008, pp. 031101–1–9.

[10]Allison, J. and Nazari, S., "Combined Plant and Controller Design Using Decomposition-Based Design Optimization and the Minimum Principle," *In the Proceedings of the 2010 ASME Design Engineering Technical Conference*, ASME, Montreal, Quebec, Canada, 2010.

[11]Allison, J. and Han, Z., "Co-Design of an Active Suspension Using Simultaneous Dynamic Optimization," *In the Proceedings of the 2011 ASME Design Engineering Technical Conference*, ASME, Washington D.C., U.S.A., 2011.

[12]Giffin, M., de Weck, O., Bounova, G., Keller, R., Eckert, C., and Clarkson, P. J., "Change Propagation Analysis in Complex Technical Systems," *Journal of Mechanical Design*, Vol. 131, No. 8, 2009, pp. 081001–1–14.

[13]Collopy, P. and Eames, D., "Aerospace Manufacturing Cost Prediction from a Measure of Part Definition Information," *Society of Automotive Engineers*, 2001, SAE Technical Paper 2001-01-3004.

[14]Mattila, J. and Virvalo, T., "Energy-Efficient Motion Control of a Hydraulic Manipulator," *In the Proceedings of the ICRA '00, IEEE International Conference on Robotics and Automation*, Vol. 3, 2000, pp. 3000–3006.

[15]Frischknecht, B. D., Peters, D. L., and Papalambros, P. Y., "Pareto Set Analysis: Local Measures of Objective Coupling in Multiobjective Design Optimization," *Structural and Multidisciplinary Optimization*, Vol. 43, No. 5, 2010, pp. 611–624.

[16]Field, G. and Stepanenko, Y., "Iterative Dynamic Programming: an Approach to Minimum Energy Trajectory Planning for Robotic Manipulators," *In the Proceedings of the 1996 IEEE International Conference on Robotics and Automation*, Vol. 3, 1996, pp. 2755 –2760.

[17]Li, Y. and Bone, G., "Are Parallel Manipulators More Energy Efficient?" *In the Proceedings of the 2001 IEEE International Symposium on Computational Intelligence in Robotics and Automation*, 2001, pp. 41–46.

[18]Sato, A., Sato, O., Takahashi, N., and Kono, M., "Trajectory for Saving Energy of a Direct-Drive Manipulator in Throwing Motion," *Artificial Life and Robotics*, Vol. 11, 2007, pp. 61–66.

[19]Spong, M. W., Hutchinson, S., and Vidyasagar, M., *Robot Modeling and Control*, John Wiley & Sons, Inc, 2005.

[20]Papalambros, P. and Wilde, D., *Principles of Optimal Design: Modeling and Computation*, Cambridge University Press, Cambridge, UK, 2nd ed., 2000.

[21]Gill, P., Murray, W., and Saunders, M., "An SQP Algorithm for Large-Scale Constrained Programming," Tech. rep., Systems Optimization Laboratory, Department of Operations Research, Stanford University, 1997.

[22]Audet, C. and Dennis, J., "Mesh Adaptive Direct Search Algorithms for Constrained Optimization," *SIAM Journal on Optimization*, Vol. 17, No. 1, 2006, pp. 188–217.

[23]Pitti, A. and Lungarella, M., "Exploration of Natural Dynamics Through Resonance and Chaos," *Proceedings of the 9th Conference on Intelligent Autonomous Systems*, 2006, pp. 558–565.

[24]McGeer, T., "Passive Dynamic Walking," *The International Journal of Robotics Research*, Vol. 9, No. 2, April 1990, pp. 62–82.

[25]Collins, S., Ruina, A., Tedrake, R., and Wisse, M., "Efficient Bipedal Robots Based on Passive-Dynamic Walkers," *Science (New York, N.Y.)*, Vol. 307, No. 5712, Feb. 2005, pp. 1082–5.

[26]Williamson, M., "Oscillators and Crank Turning: Exploiting Natural Dynamics With a Humanoid Robot Arm," *Philosophical Transactions of the Royal Society A*, Vol. 361, No. 1811, 2003, pp. 2207–2223.

[27]Ahmadi, M. and Buehler, M., "The ARL Monopod II Running Robot: Control and Energetics," *In the Proceedings of the 1999 IEEE International Conference on Robotics and Automation*, Vol. 3, 1999, pp. 1689 –1694.

[28]Allison, J., "Plant-Limited Co-Design of an Energy Efficient Counterbalanced Robotic Manipulator," *In the Proceedings of the 2012 ASME Design Engineering Technical Conference*, ASME, Chicago, IL, U.S.A., 2012.